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PRELIMINARY EVALUATION OF EARTH TARGETS FOR USE IN IMPACT EFFECTS STUDIES

Sandia Corporation
Albuquerque, New Mexico

DEVELOPMENT

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Sandia Corporation
Albuquerque, New Mexico

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Approved by: V. E. Blake
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ABSTRACT

The Aerospace Nuclear Safety Program at Sandia Corporation requires an independent assessment of the safety of all nuclear power supplies used in aerospace under all conditions from factory shipment through re-entry from space. The effects of impact against any point on the earth by a nuclear power supply or its components must be evaluated as part of this assessment. This report: (1) evaluates the fundamental mechanisms involved in soil impacts by cylindrical projectiles with velocities of 500-700 feet per second; (2) defines the problem of specifying impact targets which closely simulate various portions of the earth's surface; and (3) describes an analytical and experimental program intended to obtain the information needed to properly specify a simulated earth target for use in Aerospace Nuclear Safety impact tests.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

PRELIMINARY EVALUATION OF EARTH TARGETS FOR USE IN IMPACT EFFECTS STUDIES

I. Introduction

The Aerospace Nuclear Safety program at Sandia Corporation requires an independent assessment of the safety of all nuclear power supplies which are to be used in aerospace applications. This safety assessment must cover all credible hazards which might occur under any conditions that could exist from the time of factory shipment through re-entry from space. One major condition which requires analytical and experimental investigation is that of power supplies and/or their components impacting any point on the surface of the earth at varying velocities.

In order to provide data and information of value for assessing the safety of nuclear aerospace power supplies, the ground tests must be cogent, engineering-type tests which are so well planned and performed that test conditions can be closely controlled to simulate actual conditions. In the case of earth impact tests, conditions of velocity, temperature, attitude, and structural strength of the test item at impact must be closely controlled. These requirements obviously preclude actually dropping a test item from an aircraft against the natural earth because it is impossible, practically, to sufficiently control the conditions of test item attitude, temperature, and velocity during a free-fall drop test to produce the required cogent data. The only recourse remaining is to conduct earth impact tests at a test laboratory location where the test conditions can be closely controlled and to use a test target that closely simulates several representative areas of the earth's surface.

A survey of impact targets used in previous tests to simulate the earth's surface reveals that these targets have not closely simulated the earth's surface. Recognizing this situation, Sandia Corporation has launched a program to evaluate the requirements of an earth target and to develop a specification for a series of such targets. This report discusses this program.

II. Statement of Program Objective

The program described in this report will evaluate the requirements of an earth impact target and develop a specification for a series of impact targets which will adequately simulate the variety of earth surfaces that might be impacted by a nuclear power supply intended for aerospace applications.

The attainment of this objective implies a program consisting of the following phases:

1. A general, analytical study of fundamental mechanisms involved in the penetration of earth media (ranging from solid rock to water) by cylindrical objects traveling at 500-700 feet per second.

2. Studies of model projectile penetrations into various media to empirically relate these laboratory results to the results of the analytical study.

3. Full-scale studies of vertical projectile penetrations into various simulated earth media to empirically relate the results observed to the results of previous model studies.

4. A preliminary, full-scale study of horizontal projectile penetrations into various simulated earth targets to empirically relate results observed to those of previous full-scale, vertical penetration studies.

5. Preparing a tentative specification for a simulated earth target which will be used for horizontal rocket sled tests.

6. An experimental, full-scale study of projectiles penetrating horizontally into various simulated earth targets which are fabricated to the previously prepared tentative specification.

7. Preparing a final specification which defines earth targets for horizontal rocket sled tests which closely simulate actual earth impacts.

Various corollary studies (such as model similitude, water impact mechanisms, earth viscosity variations, and compressibility and stress-strain-time relationships for various soils under varying rates of strain) will also be performed to help achieve the objectives of this program.

III. The Earth's Surface as a Target Material

It is possible for an aerospace nuclear power supply or some of its components to impact upon any part of the earth's surface during certain conditions of launching and of flight duration. To completely assess the safety of these aerospace units, the effects of such impacts upon the earth's surface must be evaluated. Ground impact tests to evaluate these effects require targets which simulate representative areas of the earth's surface.

The surface of the earth is far from being a homogeneous and isotropic medium. It ranges from very hard exposed rock through many varieties of soil cover to water. In a very few feet, the properties of the solid portions of the earth's surface can vary greatly in both horizontal and vertical directions. Although soil maps and geological maps have been made of many parts of the surface of the earth, much of the total land area has not been accurately mapped. The U. S. Corps of Engineers and other agencies are currently mapping uncharted surface areas all over the world. When and if this mapping is completed, it may be possible to statistically predict the probability of an aerospace power unit impacting a particular type of surface. However, in order to evaluate the effect of a power plant impacting a given type of surface it will be necessary to perform impact tests against a limited number of simulated earth targets which have been arbitrarily selected as typical of the various kinds of earth surfaces.

Although it is possible to classify the surface of the earth as rock, soil, or water, in nature these materials are blended to form a varying medium. Even uniform media exhibit variations in strength. For example, the harder igneous rocks, such as basalt and felsite, have the highest potential shear strength, but even these rocks have joints and cracks which weaken the material non-uniformly.

Soil is composed of water, air, and mineral particles. Its strength and in situ modulus of deformation depend on: (1) the type, shape, and size of the mineral particles; (2) the structural arrangement of the mineral particles; (3) the moisture content; (4) the density; (5) the original confining pressure; (6) the time rate of loading; and (7) the stresses caused by the loading element. The ultimate strength of a soil also depends upon the size of the loading element and is highly time dependent.

Soils range from sands and gravels which have low void ratios (the ratio of the volume of the voids in a soil to the volume of the solids) to very fine, sensitive clays with high void ratios whose moisture content may vary as much as several hundred percent. In most soils, the shear strength increases as the void ratio and moisture content are reduced; however, this relationship is not always true.

No such thing as a "typical soil" exists in nature. Therefore, one must arbitrarily select a limited number of soil types to be simulated so that the effects of impact upon any soil can be reasonably predicted. Unfortunately, the proper soil types for simulation cannot yet be selected without more information. However, the acquisition of this information will be one of the results of this entire program. To limit soil variations for better understanding, the earth's surface might be classified into several arbitrary groupings of varying shear strength.

Rock	1.	Massive igneous rock - basalt, diabase, felsite, etc.
	2.	Stratified limestone
	3.	Coarse gravel and fragmented material
Sand	4.	Dense sand with low moisture content
	5.	Loose blow sands
	6.	Saturated dense sands
Silt	7.	Dry cemented loose silts or loess
	8.	Saturated silts
Clays	9.	Desiccated clays
	10.	Overconsolidated saturated clay
	11.	Normally consolidated saturated clays
	12.	Water

NOTE: These groups are not necessarily arranged in order of descending resistance to penetration or shear strength, but are arranged in order of descending size of the particles making up the media.

In each of the above groups, the lateral and vertical pressures increase with depth. For earth pressure at rest, there are no shear stresses in the horizontal and vertical planes. By definition, the value of the lateral earth pressure existing in the undeformed mass of natural soil with a level surface is called the "lateral earth pressure at rest."

As can be seen, the earth's surface is a complex target presenting media of varying strength to the impact of an incoming projectile.

This report discusses the mechanism of penetration in the various media outlined above, in order to develop criteria for selecting a limit number of "typical" soils to study the impact of nuclear power supplies in free fall on the earth's surface.

IV. Simulated Earth Targets Presently Being Used

A. Description of Present Targets

For many years, simulated earth targets have been used in horizontal rocket sled impact tests at many laboratory locations. In fact, several impact tests of aerospace nuclear power supplies have been performed using simulated earth targets. As explained earlier, the use of horizontal rocket sled tracks is highly desirable for these tests because a variety of test conditions can easily be controlled closely to insure meaningful results.

Earth targets at Sandia Corporation consist of compacted sandy silt with a small, undetermined amount of colloidal clay. The material is excavated from its natural site near the Sandia sled track. After being moistened, it is compacted in approximately 6-inch-thick layers by a hand-held air-powered compactor into an 8-foot x 8-foot x 8-foot wooden form at approximately 95 percent of Standard A. A. S. H. O. Density. The unit weight of the excavated material and the moisture content of the compact material are unknown.

The dimensions of the target are selected so that the test item, after impact, is completely embedded and has come to rest before the shock wave it initiates reaches the outer surface of the target and disintegrates it. High-speed motion picture films of past tests reveal that the test item does come to rest before the target is disrupted.

As presently used, the earth target is free-standing above the surface of the ground. The sides are not confined or restrained in any manner. No attempt is made to reproduce the normally in situ stressed condition of the soil when it is remolded into the target.

B. Possible Inadequacies of Present Targets

The ultimate shear strength of a soil depends on: (1) the constitutive properties, (2) the effective mean pressure, and (3) the rate of strain. The constitutive properties include: (a) particle type, size, and shape; (b) soil structure and degree of cementation; (c) void ratio; and (d) moisture content. The effective mean pressure depends on: (a) the boundary conditions, (b) initial porewater pressure, (c) stresses caused by the penetrating item, (d) the rate of drainage and (e) stress history.

The total lateral pressure of the earth at rest is very difficult to measure, but it is approximately equal to 0.4 to 0.7 times the vertical pressure which is the total weight of the material above the point being considered. Effective pressure is equal to the total pressure minus the porewater pressure. The porewater pressure at a point acts equally in all directions. In the case of normally consolidated plastic clays, the lateral earth pressure might be very nearly equal to the vertical pressure because of creep and stress relaxation with time.

If the surface being considered is at rest, there are no shear stresses on either the vertical or horizontal planes and the vertical and lateral pressure increases with depth.

Present earth targets have no external confining pressure increasing with depth, but are held in place solely by internal forces or cohesion. It is the cohesive strength (which is a function of: (1) moisture content, (2) particle type and orientation, (3) void ratio, and (4) stress history) that allows the present target to stand vertically. If the target were constructed to some greater height, it would fail in shear because of its own weight once the forms were removed. If the target material had only frictional strength, such as a dry or saturated sand, it would fail in shear once the forms were removed regardless of the height. In that case, the angle of inclination of the faces of the cube would approach the angle of internal friction of the material. After the forms are removed, shear resistance is mobilized along the base of the cube. The weight of the material causes the sides to bulge and the top of the cube to subside. This process continues until enough cohesive and frictional resistance is mobilized within the mass to keep the cube in equilibrium. The cube is strained throughout, but the strains are most pronounced in areas behind the faces of the cube. This is usually referred to as the slope stability problem. The soil target cube is in this strained state, with much of its shearing resistance already used up before the projectile begins its penetration.

Therefore, it can be concluded that the boundary conditions of the present targets are in no way the same as those existing in the same material located in situ in nature. Likewise, the modulus of deformation and other fundamental soil parameters are not the same, because their values depend upon the external boundary conditions. One could not expect the results from impact tests performed with these targets to correspond to the results achieved from impacting a test item on identical material in its natural location, unless the boundary conditions in the target are made to equal those of the earth at rest.

In addition, if the materials used in the target were compacted to exactly the same density and moisture content as the natural soil in its unexcavated location (a situation which is far from true) these materials still would not have the same shear strength, even though the boundary conditions had been identical. This is because the excavated material has been extensively remolded and its structure greatly changed. It is, in effect, a completely different material after disturbance from that material which existed before it was excavated from its natural location.

V. Hypothesis of a Penetrating Mechanism

A. General

This section of the report presents a hypothesis of a possible mechanism for a cylindrical projectile penetrating into an earth medium. For this hypothesis, the example of the impact of a SNAP 10A core vessel as a projectile is used.

At the moment of impact the projectile is assumed to be traveling at terminal velocity in the air with no horizontal velocity component. The projectile weighs 150 pounds and the impact velocity is 550 feet per sec; the diameter is 10 inches and the length 15 inches. The total energy (E_T) is equal to the sum of the kinetic energy (E_K) plus the potential energy (E_P). The total energy that must be dissipated before the projectile comes to rest is:

$$E_T = E_K + E_P = \frac{MV^2}{2} + Wd,$$

where

M is the mass of the projectile $= \frac{150 \text{ lb. force}}{32.21 \text{ ft/sec/sec}} = 4.66$ slugs

W is the weight of the projectile = 150 lb. force
V is the velocity at impact = 550 ft/sec
d is the depth of penetration in feet.

Thus

$$E_T = \frac{(4.66 \text{ slugs})\ (550'/\text{sec})^2}{2} + (150 \text{ lbs.})\ (d \text{ ft})$$

$$= 704,362 \text{ ft. lbs.} + 150\ (d) \text{ ft. lbs.}$$

B. Non-Penetrating Projectile

In the case where the projectile does not penetrate the surface, rebounds, or is destroyed upon impact, the energy will be dissipated in several ways, by:

1. Sudden generation of heat such as a flash of light and sparks. This energy loss may be insignificant but triboluminescence and frictional heat might be studied to evaluate the magnitude of this energy dissipation.

2. Generation of sound wave caused by the impacting projectile. This energy loss is probably insignificant also, but some study can be made of its magnitude.

3. Rise in temperature of the projectile and the soil. There will be a loss of energy because of the compression of the projectile and the compression of the soil surface. Some guess can be made about the pressures involved, and the magnitude of this energy can be estimated.

   The heat loss is probably not recoverable, but some of the energy used to compress the soil and projectile will be recovered if the projectile does not penetrate, and this energy will then accelerate the projectile back into the air. This same energy may also deform, rupture, or fracture the projectile.

4. Compression of the projectile and soil will cause some rise in temperature of both soil and projectile and is not recoverable. Some of the energy used to compress soil and projectile will produce the force to rebound the projectile or its parts into the air. If the force caused by the deceleration of the projectile produces stresses beyond the elastic limit in the metal, the projectile may be deformed or ruptured; this energy is not recoverable. Similarly, for the soil, energy may be used to comminute, deform or shear sufficient soil at the surface to form a crater. This energy is not recoverable to rebound the projectile.

5. Generation of at least four kinds of soil waves. At the instant of impact, four types of soil waves will be radiated out from the point of impact. The body waves which radiate spherically are the compression wave and the shear wave. The compression wave oscillates in the direction of propagation. The other two waves are surface waves and are called the Love wave and the Rayleigh wave. (Two other waves called the coupled wave and the hydrodynamic wave may also exist, but are thought to be of minor importance.) The Rayleigh wave causes the soil surface to ripple much like a wave on water except that the movement of the particles is elliptical with the long axis perpendicular to the surface of the earth, and in the direction upward and toward the point of impact. The Love wave is a surface shear wave. The particles move in a path parallel to the surface of the earth and also at right angles to the direction of travel.

The velocity of propagation of these waves varies approximately as the square root of the density, and the period increases with the distance from the origin. The frequency is believed to be approximately proportional to the velocity cubed divided by the distance away from the point of impact. The velocity of propagation for loose dry soils is thought to be about 600 to 3,000 ft/sec; for compact, saturated sands and gravel 3,500 to 5,500 ft/sec; for clay 5,000 to 6,000 ft/sec.

Studies perhaps can be made of the energy dissipated by these soil waves; at least it should be possible from existing data to make some reasonable estimate of the loss.

C. Penetrating Projectile

If the projectile is rigid enough and the soil shear strength sufficiently low, the projectile will not be destroyed nor will it rebound; instead it will continue to penetrate the soil by shearing, comminution or pulverization and compression of the soil. Energy would still be lost by the phenomena delineated above. Energy will also be lost if the soil near the surface is sheared and given sufficient velocity to form a crater around the penetration hole. Nonsaturated soils can certainly be compressed, but saturated soils are essentially incompressible during rapid loading since the water does not have time to drain out. The water and individual soil particles are almost incompressible. If the soil has sufficient cohesive strength, the penetration hole remains open after impact; thus, if the soil is incompressible, the displaced volume of the soil once occupying the hole must flow around the projectile toward the surface and raise the level of the surface. Energy, of course, is dissipated in this operation. For soils such as dry sand and gravel with no cohesive strength, the penetration hole probably does not remain open and the surface is raised by: (1) the throw out, (2) the volume of the projectile itself, and (3) any dilation of the soil caused by the shearing deformation. If the soil is loose and below critical density, the dilation is negative. If the sand is saturated and the density is greater than critical, cavitation will have to occur between the soil particles. It is doubtful that there can be any pulverization of silts and clays because these particles are already small and plate-like. Silt particles range in thicknesses from 0.06 mm to 0.002 mm and clay particles have thicknesses from 0.002 mm down to about 7.5 Angstroms. If these soils were not saturated, compression would result from structural collapse. Rock, gravel, and sand would be ground into finer particles by the energy of the impacting projectile if they are not saturated and if the gravel and sands are above critical density. If these large grained soils are comminuted, the void ratio would increase. The soil shock waves which are initiated when the projectile hits the surface will probably collapse the structure in the loose, unsaturated materials and decrease the angle of internal friction in all saturated and in all dense materials. In all cases, the wave fronts will precede the projectile by a very large distance, since the projectile even at impact is not traveling at the speed of sound.

Because of the diversity of soils and the many different types of phenomena taking place during shearing, the problem should be considered by studying at least four different general types:

Type 1. Saturated Loose Sand and Silts and Saturated Normally Consolidated Clay

a. These materials have a drained strength that is higher than the undrained strength.

b. When sheared, the volume of these soils tends to decrease.

c. No drainage can occur because the event takes place in such a short length of time that the material is essentially incompressible and at constant volume.

d. The soil structure is tending to decrease in volume upon shearing, and the soil is saturated; thus high compressive porewater pressures result. The high pore pressure reduces the total pressure between particles and causes the effective pressure to be small.

e. For a noncohesive soil, the frictional shear strength in a plane is equal to the effective pressure on that plane times the tangent of the angle of internal friction. If the effective pressure approaches zero, this material then should behave as a viscous liquid. This condition is sometimes referred to as 'liquefaction'.

For cohesive normally consolidated clay with little frictional strength originally, the cohesion should be reduced markedly because of the high pore pressure and it too should behave as a viscous liquid. This reduction in strength is probably a function of the moisture content of the soil.

Of course, for both soils the viscous shear resistance will increase as the rate of strain increases. The general behavior pattern should resemble a non-Newtonian liquid and might be described by the Bingham model of a non-Newtonian liquid.

Type 2. Saturated Dense Sand and Silt and Saturated Overconsolidated Clays

a. These materials have an undrained strength greater than the drained strength. For clays this is only strictly true so long as the mean of the principal stress is lower than the preconsolidation load.

b. When sheared, the volume of these soils tends to increase.

c. As in the Type 1 conditions, no drainage has time to take place and the material is essentially at constant volume.

d. The soil's structure tends to increase in volume, but the only way this can happen is for voids to form in the media. This condition is referred to as soil cavitation.

e. This cavitation causes high tensile stresses in the water and increases the effective stresses between the particles; thus the frictional strength increases as the rate of shear increases for noncohesive materials. There should not be a very large increase in the cohesive strength when purely cohesive overconsolidated clay is sheared dynamically.

In each of these soils the behavior should be somewhat similar to that of an elastic solid, with the strength a function of the rate and amount of strain.

Type 3. Dense, Dry to Moist Sands, Silts, and Clays

a. It is very difficult to evaluate the initial effective stresses with these soil masses because the material is not saturated; therefore, discussion of the strength of these materials is usually based on total stresses.

b. When sheared, the volume tends to increase and any water in the soil is put into additional tension.

c. Because of the low moisture content the clays are near their shrinkage limits and have very high cohesive strengths. Desiccated clay should behave as a brittle elastic solid.

d. Shock waves should have little effect on the cohesive strength of the shrunken clays, but the rate of strain should influence the cohesive strength. No locking or communition would be expected.

e. Shock waves in the dense granular material should decrease the strength of material, breaking down any water meniscuses between the grains and causing the particles to vibrate. The friction strength is directly proportional to the confining pressure.

f. The frictional strength will consist of resistance to sliding and slipping and resistance to volume change. The grains are interlocked and must roll over one another as shear stresses displace them. If the confining pressure is great enough, the grains will be sheared and broken. This phenomenon is sometimes referred to as communition and gives great strength to these materials.

g. These granular materials then should behave somewhat like an elastic solid until crushing starts. The crushing phase should resemble the behavior of an ideally plastic solid until interlocking is once again brought into play. For very high loads this process can repeat itself several times.

h. The clays should behave like an elastic solid until slippage occurs, then behave as a plastic solid.

Type 4. Loose, Dry to Moist Sands and Silts

a. These materials all decrease in volume when sheared and are compressible.

b. The tension in any water in the soil is reduced.

c. The fluid within the voids is predominately air.

d. Shock waves should decrease the frictional strength of sand and collapse the structure of silt which may be cemented in its loose state.

e. The compressive component of the stress caused by the penetrating projectile as well as the shearing stresses can also cause structural collapse. It may be possible that all these phenomena will cause the volume to decrease to the critical volume for the particular stress level.

f. If the structure collapses, the shear strength may be relatively unimportant; rather, the resistance to compression will be the most important characteristic of these soils to resist penetration.

g. After the soil is compressed in front of the projectile, shear resistance will come into play when the soil reaches its critical void ratio. This shear resistance should be a constant for a given rate of strain and given mean principal stress.

D. Proposed Trial and Error Solution

In the case of a soil which is not significantly compressible, it may be possible to use a trial and error solution to determine the depth of penetration of a projectile. In order to see what the mechanism of failure is when a projectile penetrates soil, a series of crude model tests were performed on bulked silt and nearly dry ottawa sand. A projectile was held up about 6 inches above the surface of the soil and driven into the soil by striking a rod attached to the projectile. The soil was placed in a plexiglass box 2 inches thick and 2 ft. square. The dry unit weight of the sand was 108 lbs/cu. ft., and moisture content was 3 percent. The dry unit weight of the silt was 75 lbs/cu. ft., and the moisture content about 6 percent. The projectile was 1 inch wide, 2 inches long and 2 inches deep. The rod attached was 3 feet long. The weight of the steel block was 1.02 lbs., and the weight of the rod was 2.08 lbs.

Tests were also run on saturated sand and saturated clay but these results are not shown in this report.

Motion pictures were taken of each test. The time increment for the bulked silt was 2 milliseconds and the time increment for the sand was 2.5 milliseconds. Figures 1 through 6 show individual frames taken of the entering projectile in the sand and in the silt. The grid in the soil is on 2 inch centers.

The crude, qualitative, two-dimensional model tests show a crater formed at the surface and a cavity formed around the projectile throughout penetration for water, dense sand, and bulked silt. A cone of dense soil forms in front of the projectile in dense sand and bulked silt and the rate of compression of the bulked silt can be observed. There is apparently no soil contact on the sides of the projectile. It should be noted that the small thickness of the soil model and the semi-rigid plexiglass boundaries will influence the results to some degree.

Depending on the nature of the cone formed at the front of the projectile, heat and ablation of the reactor core may or may not take place. If the cone behaves essentially as a solid, fixed mass, there will be frictional heating only of soil-on-soil; and, in the brief instant of the deceleration, none of this generated heat can reach the vehicle.

On the other hand, if the soil originally forming this cone slides over the projectile surface and is replaced by other soil during the deceleration period, the possibility of soil-on-metal frictional heating exists with consequent heating of the projectile.

The total heat transfer to a semi-infinite solid may be expressed,

$$Q = 2\,k\,A\,\sqrt{\theta/\pi\alpha}\,\left(t_1 - t_i\right)$$

where

Q = heat transfer, Btu

k = thermal conductivity, (k = 10 Btu/hr ft^{o}F for 18-8 stainless steel)

A = heat transfer area, sq feet

θ = elapsed time since heating began in hours

α = thermal diffusivity, ($\alpha \equiv k/\rho c_p$ = .172 ft^2/hr for 18-8 stainless steel)

t_1 = surface temperature, ^{o}F

t_i = initial temperature of vehicle before entering soil, ^{o}F

C_p = constant pressure specific heat, B/lb in ^{o}F

ρ = density, lb in/ft^3

For ablation to take place, the projectile surface in contact with the sliding soil must exceed a temperature of approximately 2500^{o}F. If we treat the projectile as a semi-infinite solid (an easily justified assumption), use the properties of 18-8 stainless steel, use a heat transfer area ten inches in diameter, assume that $(t_1 - t_i)$ is approximately constant at 2500^{o}F and that the deceleration period is in the order of one second, then the above equation yields a value of Q = 500 Btu = 389,000 ft. lbs. This represents over 50 percent of the original kinetic energy of the projectile and would result in the removal of only six mils from the face of the reactor core. Thus, while the above figures represent only the roughest of calculations, if future tests reveal even small amounts of ablation, it would appear safe to conclude that a substantial fraction of the projectile's original kinetic energy is lost in the immediate area of its front face. Because of the complexities of this problem, this subject will not be pursued further in this evaluation.

If the energy loss by the five items listed on pages 9 and 10 can be accounted for, the remaining energy E to be used for penetration is

$$E = E_T - E_R$$

where E_R is the sum of the energy lost by:

1. Sudden generation of heat
2. Generation of sound waves
3. Rise in temperature of projectile and soil
4. Compression of projectile and soil
5. Generation of four kinds of soil waves

and where E_T is the sum of the kinetic energy plus the potential energy.

As previously calculated, E_T = 704,371 ft. lbs. + 150(d) ft. lbs. where d is depth of penetration. Therefore, the remaining energy to be used for penetration is E = 704,362 + 150d - E_R ft. lbs.

This energy E can be equated to the force P, which resists the projectile at various levels, times and distance traveled

$$E = Pd.$$

The force P resisting the projectile is a variable and is a function of the velocity, depth, angle of internal friction,cohesion, initial effective principal stresses in soil, and size of projectile. It may perhaps be a function of other parameters also. The resisting force P for a given level of penetration and velocity can also be equated to the mass of the projectile M times the deceleration a in the incremental distance Δs.

$$P = Ma$$

which is also

$$P = \frac{W}{g}\frac{\Delta V}{\Delta t} = \frac{W}{g}\left(\frac{V_1 - V_2}{\Delta t}\right)$$

where

ΔV is the change in velocity V_1 to velocity V_2 during the change in time Δt,

W is the weight of projectile, and

g is the gravitational constant.

The sum of incremental losses in energy ΔE caused by penetration in incremental distance Δs can be written

$$\Sigma \Delta E = E = \Sigma\, P\, \Delta s$$

where

$$E = (704,362 + 150d, - E_R) \text{ ft. lbs.}$$

$$\Delta s = \Delta t \left(\frac{V_1 + V_2}{2}\right)$$

and

$$d = \Sigma \Delta s$$

Therefore,

$$E = 704,341 + 150d - E_R = \Sigma P \Delta s = \frac{W}{g} \Sigma \left(\frac{V_1 - V_2}{\Delta t}\right) \Delta t \left(\frac{V_1 + V_2}{2}\right) = \frac{W}{2g} \Sigma \; V_1^2 - V_2^2$$

This problem can be solved by triai and error if the resistance to penetration P can be computed for each given type of soil for all depths of penetration and velocities, and if the sum of energy losses at the surface E_R can be evaluated. If these values can be computed the two unknowns in the above equation are Δs and ΔV.

The velocity V_1 at the surface is known. If the resistance P_1 at the surface can be computed for the known velocity, a small change in velocity ΔV_1 can be assumed. Then the incremental depth of penetration Δs_1 can be computed

$$P_1 \Delta s_1 = \frac{W}{2g} \left(V_1^2 - V_2^2\right)$$

$$\Delta s_1 = \frac{W}{2gP_1} \left(V_1^2 - V_2^2\right)$$

Because $\Delta V1 = V_1 - V_2$, the velocity V_2 can be computed for the level Δs_1 below the surface. The depth and velocity are now known. Therefore, the proper procedure would be: (1) to compute a new resistance to penetration P_2 which is a function of depth Δs_1 and V_2; (2) then assume a new change in velocity ΔV_2; and (3) compute Δs_2.

This procedure could be repeated until the change in velocity ΔV is zero or the final velocity V is zero.

In order to check the assumptions of ΔV, the total energy lost by penetration must be found equal to the summation of the penetration resistance P times the incremental distance traveled.

$$E = \Sigma P \Delta s$$

where

$$\Sigma \Delta s = d$$

If the term of the resistance to penetration P times the incremental depth traveled Δs is not equal to the total energy which can be lost to penetration, the procedure would have to be repeated with smaller values of ΔV until the error is satisfactorily reduced.

From these computations, theoretical time vs depth of penetration, time vs velocity, and time vs deceleration relationships could be developed which could be checked by model and full scale tests for various soils.

This whole procedure is dependent upon computing the resistance to penetration P for each depth and velocity for a given soil.

The crude model tests indicate that the mechanism of failure for the projectile penetrating soil is similar to the mechanism of failure for a footing which is being loaded slowly. The ultimate resistance of a footing may be calculated many ways by using:

1. The theory of elasticity

2. The classical earth pressure theory

3. The theory of plasticity and by

4. The experimental results

The best of these methods is usually thought to be a modification of the Prandtl solution of the bearing capacity problem which uses the theory of plasticity.

Figure 7 is a sketch of the deformation pattern and stress zones after failure for the "static bearing capacity problem". According to Terzaghi[1] the bearing capacity P of a circular footing at depth d is given by the expression

$$P = \pi R^2 \left[1.3cN_c + \gamma d N_q + 0.3\gamma R N_\gamma\right]$$

where

P is the vertical ultimate bearing capacity of horizontal circular footings with perfectly smooth shafts

R is the radius of the footing

c is the cohesive strength of the soil

γ is the effective unit weight of the soil

d is the depth below the surface of the soil, and

N_c, N_q, N_γ are bearing capacity factors which are dimensionless and are functions of the angle of internal friction and depth.

The values of N_c, N_q, N_γ, as given by Terzaghi, are shown in the following table.

Bearing capacity factor	Angle of internal friction ϕ								
	0°	5°	10°	15°	20°	25°	30°	35°	40°
N_c	5.7	8	10	14	19	26	38	60	100
N_q	1.0	1.5	3	5	9	15	22	39	85
N_γ	0	0	0.8	2.5	5.5	12	26	45	130

[1] K. Terzaghi, Theoretical Soil Mechanics, John Wiley, N.Y.

Terzaghi's bearing capacity factors were for foundations near the surface and at shallow depths. This work has been extended by Meyerhof[2] to include foundations placed at intermediate and great depths. Generally, all the bearing capacity factors increase with depth.

The bearing capacity equation above represents an incompressible soil with boundary conditions described as earth pressure at rest, i.e., the surface of the homogeneous soil is horizonatl and extends infinitely in all directions. Of course the lateral and vertical effective pressure increases with depth. Therefore, this bearing capacity equation would not hold for a projectile penetrating laterally into a cube. The shearing strength of this soil is given by the equation $\tau = c + p \tan \phi$ (Mohr-Coulomb theory of rupture) where c denotes unit or apparent cohesion, ϕ denotes the angle of internal friction and p denotes the effective normal pressure on the shear plane.

Resistance to penetration can be computed if the cohesive strength of the soil and the bearing capacity factors are known. For a given projectile, the bearing capacity factors are functions only of depth and angle of internal friction. Cohesive strength and angle of internal friction are functions of rate of strain; therefore, it is necessary to make some assumptions about the rate of strain on the plane of failure when the projectile penetrates the soil. It seems reasonable that the velocity of strain or strain rate along the failure planes at a given depth is about the same as the velocity of the projectile at that depth. Therefore, in order to compute the resistance to penetration, it is necessary to know the relationship: (1) between the rate of strain and the angle of internal friction; and (2) between the rate of strain and cohesion. Some work has been done in this area at M.I.T.[3] and at Harvard.[4] However, additional laboratory work is required. The most promising device for developing the required relationship seems to be the simple shear apparatus developed by H. K. Roscoe at Cambridge University. Other equipment and approaches can also be used.

If the depth of penetration computed using the bearing capacity equations and the trial and error method outlined above cannot be verified by actual field and model tests, then other surfaces of failure must be assumed and new bearing capacity factors computed. Also, if in future model tests with other shapes of projectiles, other soils, and higher initial velocities, it is observed that the failure surface deviates from that assumed by Meyerhof, Terzaghi and others, new resistance formulas will have to be devised to fit the mechanism of failure. One possibility is to calculate the shear resistance at plastic failure for the projectile at various depths. This might be done by assuming various failure planes and by using the minimum value of shear resistance for each depth. Because there are an infinite number of surfaces of possible failure, this calculation appears an overwhelming task. But it may be possible by using a computer and by assuming surfaces that seem similar to the two-dimensional failure planes shown by model tests. By this same procedure it may be possible to compute the lateral penetration of projectile moving horizontally into a cube of soil, if the proper boundary conditions are used.

In a highly cohesive soil the <u>unit</u> resistance to penetration should increase as the initial cohesive strength and the rate of strain increase. However, the unit resistance should only be slightly influenced by the diameter of the projectile and the depth of penetration.

In a cohesionless soil the <u>unit</u> resistance to penetration should increase as the depth, size and velocity of projectile and as initial angle of internal friction increase.

Thus, for most soils the resistance of penetration will increase with depth. Therefore, the deceleration will increase with depth. This is just the opposite for water, because resistance is a function of the square of the velocity. If the velocity change in the soil is linear, the deceleration is constant and the resistance to penetration is constant. As the moisture content increases, one might expect the resistance to penetration to be a function of velocity and that the lower bound would be water penetration. A mathematical model devised for saturated soils should degenerate into the water entry resistance formulas when the mass density and viscosity of the soil are considered.

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[2] G. G. Meyerhof, "The Ultimate Bearing Capacity of Foundations, "<u>Geotechnic</u>, V.2, 1951, pp. 301.

[3] <u>Shearing Resistance of Sands During Rapid Loadings</u>, Report to Waterways Experiment Station (MIT, 1962)
<u>Undrained Strength of Saturated Clayey Silt</u>, Report to Waterways Experiment Station (MIT, March 1963).

[4] A. Casagrande, S.D. Wilson, "Effect of Rate of Loading on the Strength of Clays and Shales at Constant Water Content", <u>Geotechnic</u>, V.2, 1951, p. 251.

The velocity patterns for a steel block penetrating sand and bulked silt are shown in Figures 8 and 9. These data are very rough but give some indication of what can be expected.

E. Water Entry Problem

The water entry problem should be studied as the lower bound of clays with very high saturated moisture contents such as swamps and muskegs. Also more information and experimental data are available for this problem and any formula devised for projectile penetration into soils should degenerate into formulas for water entry as the density and viscosity decreases to that of water. This should happen as the moisture content of the soil increases. Therefore, a study of the liquid stage problem should result in information which might be useful in predicting the action of a saturated clay soil.

As clay is added to water, the density and viscosity of the mixture increases. The increased density tends to increase the Reynolds number and the increased viscosity tends to decrease the Reynolds number ; consequently, the drag coefficient is probably not changed significantly, and the problem can be calculated in the same way that the clear-water problem is treated. Even if these changes do not compensate each other or even if they both caused the Reynolds number to change in the same direction, the drag coefficient seems to be essentially independent of the Reynolds number at values of the Reynolds number beyond 10^4. For a Reynolds number of 10^4 and water as the medium, the 10 inch prototype body would require a velocity of less than one foot per second; but the terminal velocity in water is about 18 feet per second, so that the drag coefficient may be considered to be a constant during the entire deceleration period.

In Navord report (1308) on water entry whip and deceleration of full scale torpedo models with ogive and plate ogive heads, Waugh and Ager determined:

1. The deceleration at entry was measured photographically by analyzing the pictures from water entry to tail submergence.

2. The entry of the torpedos with a flat head was limited to a velocity of about 300 fps to 350 fps instead of the 500 fps experienced by the other shapes. From the results, the average entry decelerations over the 12 foot length of the torpedos were about 180 g's.

3. By extrapolating from 350 fps up to an entry velocity of 550 fps, the average entry deceleration over the length of the torpedo is about 450 g's. From the calculated deceleration pattern for the present problem involving the flat nosed cylinder, the acceleration pattern varies from 28,600 fps^2 at zero submergence to a value of about 3000 fps^2 at a depth of 12 feet. The average deceleration over the 12 foot length is

$$a_{avg.} = -\frac{(28,600 + 3000)}{2} = 15,800 \text{ fps}^2$$

In terms of gravitational units

$$a_{avg.} = \frac{15,800}{32.2} = 490 \text{ g's}$$

This agrees closely with extrapolated value of 450 g's taken from the experimental curves. Experimental values given in the report "Torpedo Studies" by O.S.R.D. in 1946 are also comparable to the values cited above.

The Waugh report indicated that the drag coefficient at entry is essentially the same as though the water had flowed around the cylinder, so at the surface the drag force is the force exerted on the cylinder, except that the water surface goes down and the force at entry is probably not the force calculated from the entry velocity. However, if compression wave forces are neglected, the theoretical resisting force based on entry velocity would be:

$$P = C_D \frac{\rho V^2}{2} A$$

For the 10 inch cylinder the equation becomes

$$P = (0.83)\left(\frac{1.94}{2}\right)(0.545)\ V^2 = 0.439V^2$$

$$= (0.439)\ (550)^2 = (0.439)\ (30.2)\ (10^4)$$

$$= (439)\ (30.2) = 133.3 \times 10^3 \text{ lbs.}$$

$$= 133,300 \text{ lbs.}$$

where

P = Drag force on the cylinder in lbs.

C_D = Drag coefficient

ρ = Mass density of water in slugs

A = Cross sectional area of the cylinder in square feet.

V = Velocity of the cylinder in fps.

Check:

If $a = 28,600 \text{ fps}^2$ and $P = 133,300$ lbs.

then

$$P = Ma \qquad M - \frac{133,300}{28,600} = 4.66 \text{ slugs}$$

where M is the mass of the cylinder in slugs and a is the acceleration in feet per second squared.

$$\frac{W}{g} = M = 4.66 \text{ slugs}$$

$$W = (4.66)\ (32.2) = 150 \text{ lbs.}$$

The average pressure on the face of the cylinder is then:

$$\frac{P}{A} = p = \frac{133,300}{0.545} = 245,000 \text{ lbs/ft}^2 = 1700 \text{ psi.}$$ based on the entire area of the cylinder.

Because of cavitation and separation, the effective area supporting the drag force is probably less than the 0.545 square feet, so that pressures in excess of 2000 psi are probably acting on parts of the nose of the cylinder. The drag coefficient of 0.83 is taken from Navord Report 1308 by Waugh and Ager in which they found the drag coefficient to be 0.83 for entry velocities of about 350 fps. The coefficient for the 10 inch cylinder in the present study should be about the same as for the torpedos in the Navord 1308 report.

In the book "Torpedo Studies" from the office of Scientific Research and Development, 1946, pages 50 and 51 state that the impact pressure is theoretically infinite but, because of compressibility, the actual impact pressure is only

$$p = \rho c V_e$$

$$p = (2.0)(4800)(550)$$

$$= (4800)(1100)$$

$$= 5,280,000 \text{ lbs/ft}^2$$

$$p = 36,700 \text{ lbs/in}^2$$

where

ρ = mass density of sea water in slugs

c = velocit· of a compression wave in water in fps.

V_e = Entry ve ocity of the projectile in fps.

Although these elas 'c pressures at entry are theoretically very high, if they exist in reality, they are of such short duratic 1 that they are probably not highly significant in the deceleration pattern and in the ability to do damage. Theoretically, the duration of the above calculated stress would be only a few microseconds. Howe 'er other experimenters have found that there may be significant compression wave pressures occurring at later stages of submergence. These effects might require more consideration and study in this proposed project, although it seems very unlikely that they are significant in this problem.

If the effects of compressibility are neglected, the velocity and deceleration patterns can be established by writing the simple equation of motion and solving the deceleration in terms of the velocity. This elementary approach is being used until more data are available, because all methods involve certain unproven assumptions. The basic equation used here states that the drag force minus the weight is equal to the dynamic force which is mass times acceleration. Using a drag coefficient of 0.83 and a weight of 150 pounds instead of a submerged weight because the cylinder is traveling in a vapor cavity, the basic equation

$$P - W = Ma$$

becomes

$$C_D \frac{\rho V^2}{2} A - 150 = Ma$$

and then reduces to

$$0.094V^2 - 32.2 = a.$$

This equation gives the results in the accompanying curves. (Figure 10).

The calculations show that the prototype cylinder should decelerate to a terminal velocity of 18.5 fps at a distance of 52.8 feet below the surface. If a curve of distance versus time can be established, the slope at each point on the curve represents the velocity. In turn, the slope of the velocity vs. time curve at each point represents the acceleration. On the basis of the acceleration curve it appears that the maximum forces acting on the cylinder would occur at or near the surface of the water. According to the calculations, neglecting compressibility, the approximate force on the cylinder at entry would be approximately 130,000 lbs.

To obtain experimental data for testing the method of analysis used, a small model was studied in the laboratory. A small steel cylinder weighing 0.988 lbs. was dropped into a tank of water. The cylinder was 1.5 inches in diameter and 2.05 inches long. The entry velocity of the cylinder as it entered the water was approximately 16 feet per second. Pictures were taken at 400 frames per second and the velocity of the cylinder was calculated at various depths of submergence by photographic techniques. The results of this experiment are plotted in Figure 11.

Using the method explained above, the theoretical curve was determined and plotted for comparison purposes on Figure 11. The drag coefficient used was 1.1.[1] The data from the experiment are refinements not sufficiently consistent to justify additional trial in the theoretical procedure. However, the comparison between the theory and the experimental results is fairly close, indicating that the theoretical procedure can probably be refined and used to predict the operation of the prototype cylinder striking a water surface.

F. Computing Penetration in Compressible Soils

For soils that are compressible, the Prandtl attack will probably be inadequate because the mechanism of failure is different. Such nonsaturated soils as sand, and silt in the bulked or cemented state (below critical density) are compressible at all rates of strain and fail by compression in front of and lateral to the direction of penetration. Once the structure of the soil is broken down and the density increases to critical, then shear resistance will be brought into play.

A trial and error solution can be promulgated which is similar to the one proposed in section C of this part, but compressibility of the soil and friction on the dense cone of soil on the face of the projectile will furnish the resistance. From the crude model tests (Figures 3-6), it can be seen that friction on the sides of the projectile is unimportant. The volume of soil displaced by the projectile in a given time interval will be taken up by densifying the soil, probably linearly, out to some lateral distance to form a hemisphere in front of the projectile.

The laboratory measurement of dynamic compressibility of soils might yield the distance to which the soil is influenced by the projectile and the resistance to the movement of the projectile at various velocities. If so, a trial and error solution can be obtained for the depth of penetration.

[1] Given on page 3-12, Fig. 21 of Fluid Dynamic Drag by Hoerner.

VI. Proposed Experimental Program

A. Model Tests

1. Similitude Analysis - Model tests will be run to help study this problem. In order that the results of the model tests represent the penetration of the prototype, it is necessary to consider the requirements of model similitude. In so doing, scale factors are derived from which quantitative results may be obtained.

The variables controlling this modeling problem are assumed to be:

d - penetration in feet

D - characteristic size of the projectile in feet

ρ - mass density of projectile in slugs

V - velocity of projectile in feet per second

$\tilde{V}$ - volume in cubic feet

γ - unit weight of soil in pounds per cubic foot

a - deceleration of projectile in soil in feet per second squared

c - cohesion of soil in pounds per square foot

C - compressibility of soil in pounds per square foot

ϕ - angle of internal friction of soil in degrees

g - gravitational acceleration in feet per second squared

A dimensional analysis according to Buckingham's π - theorem results in a functional equation of the form:

$$\pi_1 = f(\pi_2, \pi_3, \ldots \pi_{n-m}),$$

where

n = the number of variables involved

m = the number of dimensions included in the n variables

in which the π terms are sets of dimensionless ratios.

A dimensional analysis (see the appendix) of the variables controlling this problem results in:

$$\frac{d}{D} = f\left(\frac{D\gamma}{\rho V^2}, \frac{Da}{V^2}, \frac{c}{\rho V^2}, \frac{C}{\rho V^2}, \phi\right)$$

The dimensionless numbers for the two systems must be equal for similitude in order that the unknown function, f, shall have the same value for the model as for the prototype. The subscripts, m and p, refer to the model and prototype respectively.

Similarity between model and prototype also requires that various ratios between them be equal. That is, all geometric ratios should be based to the same ratio. Thus,

$$D_{Ratio} = \frac{Dia._m}{Dia._p} = \frac{Length_m}{Length_p} ,$$

$$\frac{Area_m}{Area_p} = D_R^{\ 2} ,$$

and

$$\frac{Volume_m}{Volume_p} = D_R^{\ 3} = \frac{\tilde{V}_m}{\tilde{V}_p}$$

All force ratios should also be equal. The ratio of dead-weight forces is obtained for the projectile:

$$F_R = \frac{W_m}{W_p} = \frac{\rho_m g_m \tilde{V}_m}{\rho_p g_p \tilde{V}_p} = \frac{\rho_m \tilde{V}_m}{\rho_p \tilde{V}_p}$$

since the gravitational acceleration will be the same for both.

The inertial force ratio is also obtained for the projectile:

$$I_R = \frac{F_m}{F_p} = \frac{M_m a_m}{M_p a_p} = \frac{\rho_m \tilde{V}_m a_m}{\rho_p \tilde{V}_p a_p} .$$

Equating the dead-weight force ratio to the inertial force ratio yields:

$$F_R = I_R = \frac{\rho_m \tilde{V}_m}{\rho_p \tilde{V}_p} = \frac{\rho_m \tilde{V}_m a_m}{\rho_p \tilde{V}_p a_p} .$$

This equality is true only if $a_m = a_p$. Thus, the deceleration for the model must equal that of the prototype.

Consider next the functional equation for the penetration, and examine first the π_3 term, $\pi_3 = Da/V^2$. For similarity, the magnitude of π_3 must be the same for both the model and the prototype. Therefore,

$$\frac{D_m a_m}{V_m^{\ 2}} = \frac{D_p a_p}{V_p^2} .$$

And since $a_m = a_p$, the relationship reduces to:

$$\frac{D_m}{D_p} = \frac{V_m^2}{V_p^2} .$$

In ratio form, this equation becomes:

$$D_R = V_R^2 . \qquad (1)$$

Equating the π_2 term, $\pi_2 = D\gamma/\rho V^2$ for the model and prototype yields:

$$\frac{D_m \gamma_m}{\rho_m V_m^2} = \frac{D_p \gamma_p}{\rho_p V_p^2} .$$

This statement may be rearranged to give in ratio form:

$$D_R \gamma_R = \rho_R V_R^2 ,$$

which may be simplified according to Equation 1 to yield:

$$\gamma_R = \rho_R . \qquad (2)$$

From $\pi_4 = c/\rho V^2$:

$$\frac{c_m}{\rho_m V_m^2} = \frac{c_p}{\rho_p V_p^2} ,$$

which in ratio form, yields:

$$c_R = \rho_R V_R^2 . \qquad (3)$$

By similar procedure with $\pi_5 = c/\rho V^2$,

$$c_R = \rho_R V_R^2 \text{ is obtained.} \qquad (4)$$

The sixth π + cr m ϕ, implies that

$$\phi_m = \phi_p \qquad (5)$$

Now that the dimensionless numbers are equal, the unknown function of the original equation for both the model and prototype is equal. Therefore, the ratio of that equation results in:

$$\frac{d_m}{d_p} = \frac{D_m}{D_p} ,$$

or in ratio form:

$$d_R = D_R, \tag{6}$$

which states that the penetration is of the same geometric ratio as the characteristic size of the projectile. To summarize, this analysis states that if Equations 1 through 5 are satisfied, the model will be similar in performance to the prototype.

2. Description of Model Tests in Soil - The tests will be begun by using the derived equations for similutude to establish the various properties of the projectile and the soil (or other media) for the model. A "model of a model" method will be used to verify the validity of the analysis and also to determine if any scale effects result from decreasing the sizes. That is, the model experiments will use two projectiles (say 3/4" and 1-1/2" characteristic dimension). Thus, the smaller one may be thought of as a model of the larger one. This is necessary because of the limited amount of data and also the difficulty of collecting that data for the full-size prototype.

The models will be two-dimensional, rather than three-dimensional as in the prototype, so behavior of the soil during penetration can be observed with the Fastax cameras. The projectiles for the models will be of parallelepiped form rather than the cylindrical form as in the prototype. The width of the model projectile will be compared to the diameter of the prototype as the characteristic dimension.

The model projectiles will be shot vertically into several types of media in various states to simulate the prototype entering different types of soils. The soil sample will be about 2 inches thick and have a horizontal dimension of about 3 feet and a variable depth depending on the material and the expected penetration. The exact size of the sample will have to be large enough so that the compression shock wave does not reach the boundary before the projectile comes to rest. These phenomena will have to be observed first by experiments before setting the final dimensions of the sample. The container of the sample will have a heavy plexiglass front so that pictures can be made during the penetration. A loose grid of carbon particles will be embedded on the face of the sample so that the movement of the sample particles may be observed.

Similar tests may also be performed by shooting the model projectiles horizontally into a soil sample. Only materials with cohesive strength or artificially imposed cohesion can be used in these tests. From these simple tests it will be seen whether or not the present horizontal rocket tests in any way compare to what will happen when the projectile impacts vertically on the earth's surface.

The two-dimensional tests will allow several important observations to be made:

(1) Mode of failure.

(2) Distortion of soil below plane of failure.

(3) Rate of progress of planes of failure.

(4) Velocity of particles forming crater.

(5) Depth and size of crater.

(6) Rate of deceleration of projectile.

(7) Amount of elastic rebound of unsheared soil.

(8) Effect of pore water.

The instrumentation and data collection will be very difficult for such items as 3, 4, 6, and 7 but the net effect should be to give a better understanding of the problem and, thus, make it easier to formulate an analytical approach to it. Three-dimensional model tests will also have to be performed to determine the factors involved in using two-dimensional rather than three-dimensional tests.

At least five different types of material will be used in the model tests. These are:

(a) Plaster of Paris to simulate the non-fissured rock.

(b) A very fine rock flour to represent the noncohesive materials.

(c) A highly plastic clay such as montmorillonite to represent the cohesive materials with the pore fluid either water or oil.

(d) Some transparent double refracting material such as gelatine for photoelastic studies with which it may be possible to actually compute the time-pressure relationship for the impacting projectile.

(e) Water.

In the case of (b) and (c), the density and moisture content can be varied over a wide range so that the effect of pore water pressure on possible liquefaction and/or cavitation and effect of changes in shear strength can be observed.

3. Description of Model Tests in Water - The problem of the projectile entering a body of water must also be considered. In order to predict the deceleration pattern, model tests will be conducted. The scale factors for similitude must be found.

As the projectile enters the water, a cavity is formed. The size and shape of this cavity will affect the path of the projectile. The Froude number, the ratio of inertia forces to gravity forces, is the most important parameter at this stage. Previous tests by other investigators have shown that, with the model open to the atmosphere and scaling factors derived according to the Froude number, good results are achieved with respect to the cavity and penetration.

The Froude number criterion requires that:

$$\frac{V_m}{\sqrt{D_m g_m}} = \frac{V_p}{\sqrt{D_p g_p}}$$

Since $g_m = g_p$, the velocity ratio must be:

$$V_R = \sqrt{D_R}\,. \tag{1}$$

To find the time relationship between the model and the prototype, the velocity may be written in terms of its basic units and combined with Eq. (1).

$$V_R = \frac{D_R}{T_R} = \sqrt{D_R}$$

Simplifying, it is seen that

$$T_R = \sqrt{D_R} \tag{2}$$

The geometric ratio between model and prototype will remain the same as previously discussed.

The model tests conducted will be similar to those conducted with soil samples. Two different sized model projectiles will be used so that the "model of a model" method may be used. From the data about the small model, predictions of the velocity and acceleration patterns will be made for the larger model. The results from the larger model will then be used to check these predictions.

Using high speed photography and accelerometers, the relationship between depth and time can be established. From this curve and from the photographs, the velocity patterns and the acceleration patterns may be determined. The areas of cavitation on the face of the projectile can also be studied and determined. From the accelerations, methods will be worked out to determine the maximum forces to be expected.

From the results of the two models, effective means of predicting the velocity and acceleration patterns can be studied. These studies can then be used to make predictions for the actual prototype.

B. Laboratory Tests

Tests on various types of earth in different states should be performed to describe the necessary empirical constants for use in the analytical solution. These tests should include:

1. Dynamic ultimate shear strength.
2. Dynamic modulus of deformation, that is, the relation between shear stress and volumetric strain, in the nonlinear elastic range, the plastic range, and in the viscous range.
3. Viscosity.
4. Dynamic compressibility.
5. Critical void ratio.

In each case the following items must be considered:

a. Time rate of strain.

b. Confining pressure or effect of depth.

c. Pore water effect on liquefaction and cavitation.

d. Type of soil with various void ratios, structure, and moisture content.

It is possible that all of these effects might be studied by using a modification of the simple shear device invented by K. H. Roscoe at Cambridge University. A cubical sample of soil is confined with various pressures and a deviator stress applied so that a state of uniform shear exists. The change in void ratio and pore pressure can be measured as well as the strain rate and load required to produce the strains. The shear stress can be applied in three different ways: (1) slowly, (2) rapidly with the use of explosive charge, or (3) with a high speed vibrator.

C. Full Scale Vertical Tests

After the model tests have been completed and some analysis finished, full scale vertical tests should be instigated. Since many horizontal tests have been performed, some full scale vertical tests should be performed so that comparison can be made between the model tests and full scale horizontal and vertical tests. If possible, at least one of the vertical tests should use a target constructed with the same material and density used in the previous full scale horizontal tests.

The targets might be constructed in an excavation adjacent to the tower that will be used to support the leads which the projectile will follow to the point of impact. This excavation should be about 10 feet square at the surface and decrease in horizontal size to about 4 feet square at depth of 6 feet, so that the target material can be compacted against the walls of the excavation to provide boundary conditions similar to ones that would exist if the target were infinite in size. Precise controls should be exercised during construction of these targets and all index properties determined for soils used in the targets. The layers should be compacted in about 6-inch lifts, so that the density and moisture content is uniform. Evaporation should be controlled and each layer scarified before the next lift is applied so that no planes of weakness will exist in the mass. The target variables will be size, boundary conditions and state of the media. If the materials are compacted into an excavation, the boundary conditions should be approximately the same as the ones that would exist where the mass is uniform over an area and depth much larger than the target. The variables which should be considered for the state of media are:

1. Type, size, and shape of mineral particles.

2. Structure of soil.

3. Density or void ratio.

4. Moisture content.

To get a fairly general picture of the behavior of the earth's surface when hit with a test item, the following type targets should be used:

*1. Concrete to simulate hard unfractured rock.

2. Dry loosely placed coarse graded sand-gravel mixture. Maximum size of aggregate 1-1/2".

3. Dense moist coarse graded sand-gravel mixture.

*4. Dense dry well graded sand.

5. Bulked moist well graded sand.

*6. Saturated dense sand.

7. Low density dry cemented silt.

8. Saturated silt.

*9. Desiccated medium plastic clay.

*10. Saturated bentonitic clay, normally consolidated if possible.

*11. Water.

If it is not possible to perform all of these tests, at least one test should be run using the target materials that are starred above.

The variables for the projectile will be the mass, size, velocity, shape, and rigidity. The shape, size, and mass should be limited to that of the SNAP reactor. Because of instrumentation difficulties, the projectile might be constructed to be very rigid. The velocity at impact should be 450, 550, and 650 ft/sec. The instrumentation for the projectile should be capable of measuring:

1. Rate of deceleration.

2. Pressure distribution over the face of projectile versus time, if possible.

A series of pressure cells, strain measuring devices, and accelerometers should be used to measure the effect of the impacting projectile on the target. Use might be made of devices now under development at the University of New Mexico. If several Fastax cameras are trained on the point of impact, the velocity of the expelled material can be measured. After the projectile is at rest, measurements should be made so

that the penetration, heave of surface, size of crater, amount of material expelled, and amount of material that falls back into the crater can be described.

D. Full Scale Horizontal Tests

Even though many horizontal tests have been performed, only one type of material has been used and little is known about its state. If the boundary condition problem can be solved, a prediction of behavior of the horizontal target and impacting projectile might be made. These predictions should be tested using several types of soil in different states. Tests of this nature would be useful also in designing a target that will simulate a given point on the surface of the earth. By comparing these horizontal tests with the vertical tests using the same target materials, it should be possible to design a system so that the boundary conditions are the same. The horizontal tests seem to be very much favored. Higher velocities and closer control of test conditions can be obtained on the long sled track than with air guns mounted in vertical towers. If the horizontal rocket sled tests are to be of much use, the horizontal targets will have to be designed so that their boundary conditions are nearly the same as the vertical targets.

Possibly a long permanent embankment should be built behind and around the target area and each target constructed against this embankment. The resistance of this embankment with proper vertical surcharges on top of the target might approximate the vertical and horizontal resistance of the earth.

The soil target might also be constructed between two steel bulkheads and pressure applied laterally. The pressure should increase from zero at the face to some maximum value at the rear of the target to simulate the increasing confining pressure of the earth. A bulkhead placed on top of the target with pressure applied in the same way would complete the boundary conditions.

In some way the face of the target must be restrained so that it is not strained by slope instability because of the weight of the material. Whatever is used will have to be expendable and not interfere with the penetration of the projectile.

Essentially this will be a trial and error procedure. A design must be generated and tested until the results are the same as the ones obtained with the vertical tests. It may very well be that different designs will have to be used for different types of material.

It might also be possible to construct a counterbalanced system in which the soil target is rotated in the plane of the sled track and the vertical direction, so that the centrifugal force is similar to the force of gravity. The boundary conditions imposed could be made almost identical to those on soil at rest. Of course it would be difficult to synchronize the rotation of the target and the movement of the rocket sled.

Once the designs are completed, specifications should be prepared for the construction of future targets which will simulate behavior of various portions of the earth when different types of projectiles are impacted at different velocities.

VII. Survey of Similar Work Being Done Elsewhere

At the outset of the Sandia Earth Target Simulation Study, it was recognized that diligent efforts must be exercised to insure that the work proposed to be done in this study will not repeat or duplicate work being done by others. At the same time, it was recognized that a close survey of other work being performed in the soil mechanics field, even though not directly connected to this study or its objectives, might yield very useful data, ideas, equipment, and techniques which could be profitably used in Sandia's study. Accordingly, conferences were held with several laboratories engaged in soil mechanics research related to the problems under study.

Several institutions are researching the basic behavior of soil under dynamic loadings, but most of this work is directed toward blast effects on underground structures, soil wave propagation, and the effect of vibratory and transient loads on individual footings. Of particular interest to this program are the stress-strain-time relationships that have been studied for various soils. The type of equipment (and the limitations of it) used in these tests is also of interest.

Some of the institutions and the areas in which they are working are listed below:

1. Massachusetts Institute of Technology, Cambridge, Mass.
   a. Dynamic Triaxial Strength Tests
   b. Seismic velocity in confined samples
   c. Soil structure interaction
2. University of Illinois, Urbana, Illinois.
   a. Footings subjected to transient loads
   b. Energy absorption of granular materials
   c. Soil structure interaction
3. U. S. Corps of Engineers Waterways Experiment Station, Vicksburg, Miss.
   a. Footings subjected to transient and vibratory loads
   b. Wave propagation in confined samples
   c. Dynamic soil structure interaction
4. U. S. Naval Civil Engineering Laboratory, Port Hueneme, Calif.
   a. Wave propagation in confined samples
   b. Footings subjected to blast loads
5. Illinois Institute of Technology Research Institute, Chicago, Illinois.
   a. Wave propagation in confined and triaxial samples
   b. Footings subjected to transient loads
   c. Dynamic soil-structure interaction
   d. Dynamic photoelasticity
6. University of New Mexico, Air Force Shock Tube Laboratory, Albuquerque, New Mexico.
   a. Dynamic soil-structure interaction studies
   b. Dynamic triaxial tests
   c. Dynamic modulus of deformation in the constrained state
   d. Dynamic shear modulus
   e. Wave propagation in cylindrical samples
   f. Static arching studies
7. Sandia Corporation, Albuquerque, New Mexico.
   a. Theoretical missile penetration studies
   b. Shock waves in solids
   c. Cratering in soils

The above list certainly does not include all the groups working in this area. There are some industrial and consulting engineering firms as well as other universities doing excellent work, but the general pattern of subjects being investigated follows those listed above.

The work concerning transient loads on footings is somewhat applicable to this study. All of this work so far reported deals with a plate placed on a confined soil and then given an impulse or blow by some sort of an actuator or a blast on the soil surface and plate. The surface deformation and depth of settlement is reported. The soil is usually dry Ottawa standard sand. This, of course, is quite a different problem from a projectile moving at high velocity and impacting on the soil, but some of the failure mechanisms are similar.

The dynamic triaxial strength tests as related to rate of strain are interesting but, like all triaxial tests, the strains and stresses are not uniform throughout the sample and it is very difficult to decipher what is truly happening to the soil on the plane of failure at the time of failure.

VIII. Time Schedule for Performance

A study such as described in this report is quite difficult to time-schedule accurately. It is very desirable to have such a time-schedule to predict the expected duration of various phases of the analytical and experimental phases of the entire study. However, it must be recognized that the analysis and determination of phenomena relating to the fundamental mechanisms of interaction between nonhomogeneous physical mediums is sometimes difficult and laborious, and time-estimates are sometimes not met.

In the light of the above statement, a proposed time-schedule for the performance of the analytical and experimental phases of the earth target simulation study has been made and is shown on Figure 13.

IX. Conclusions

Several problems have existed for a long time:

1. Determining the depth of penetration of a projectile impacting into the earth,

2. The fundamental failure mechanism involved in earth impacts of projectiles, and

3. Simulating such earth impacts with a horizontally impacted earth target.

Through the methods presented in the proposed study, it may be possible to:

1. Evaluate the present horizontal earth target's ability to simulate the behavior of the earth during impact.

2. Design horizontal earth targets that will simulate the various portions of the earth.

The proposed research may yield a mathematical model which will describe the behavior of the soil during impact. This model will depend on whether there exists a unique relationship between void ratio, mean principal stresses, shear stress, and rate of strain for the various states of a given material. There is strong evidence that such relationship does exist. These soil parameters are very difficult to measure at high rates of strain, but with proper equipment this should be possible. However, it is also possible that the mathematical model may be so complicated that only a skilled applied mathematician can solve it with the aid of a computer.

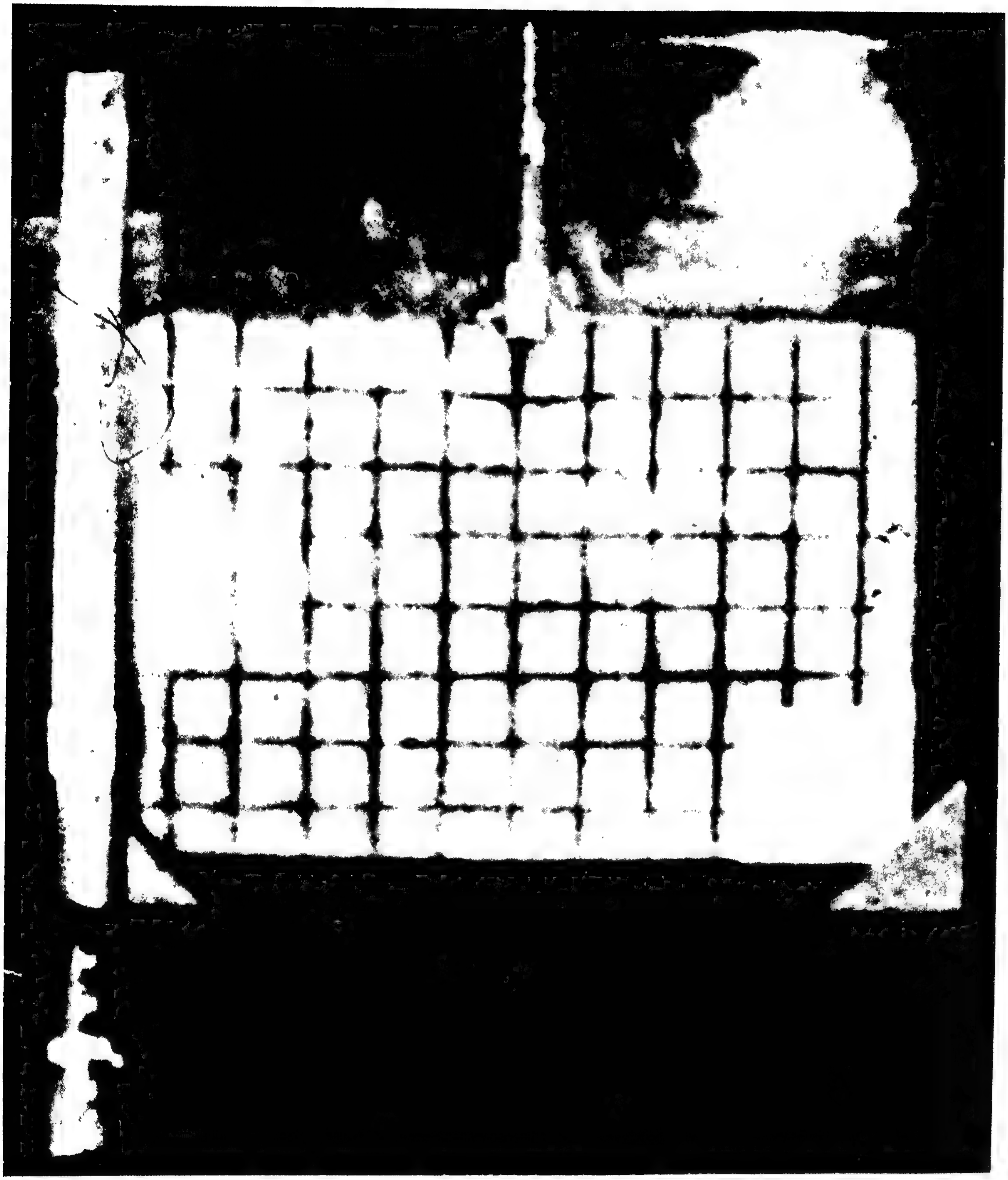

Figure 1. Preliminary Laboratory Test of Penetration Into Sand

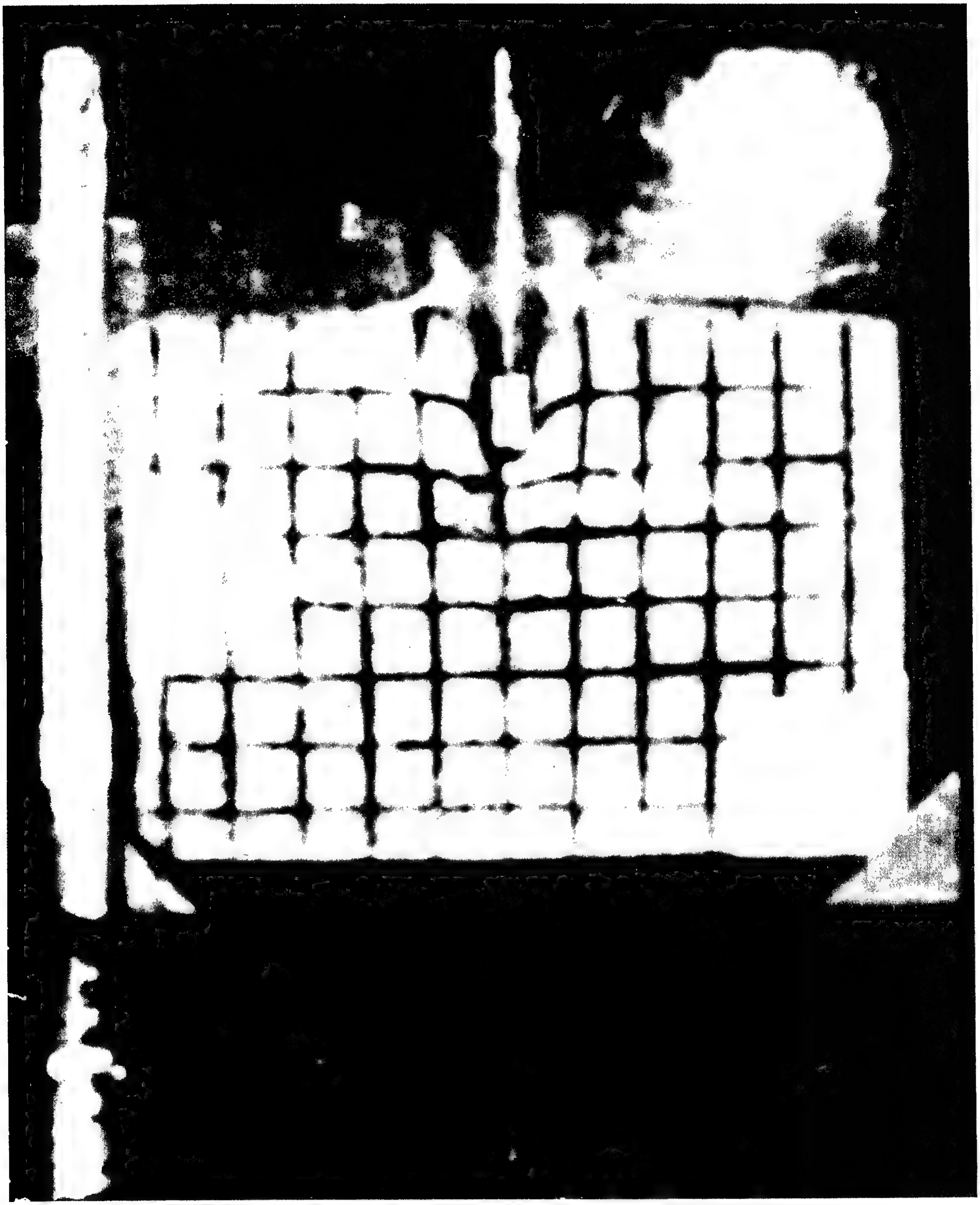

Figure 2. Preliminary Laboratory Test of Penetration Into Sand

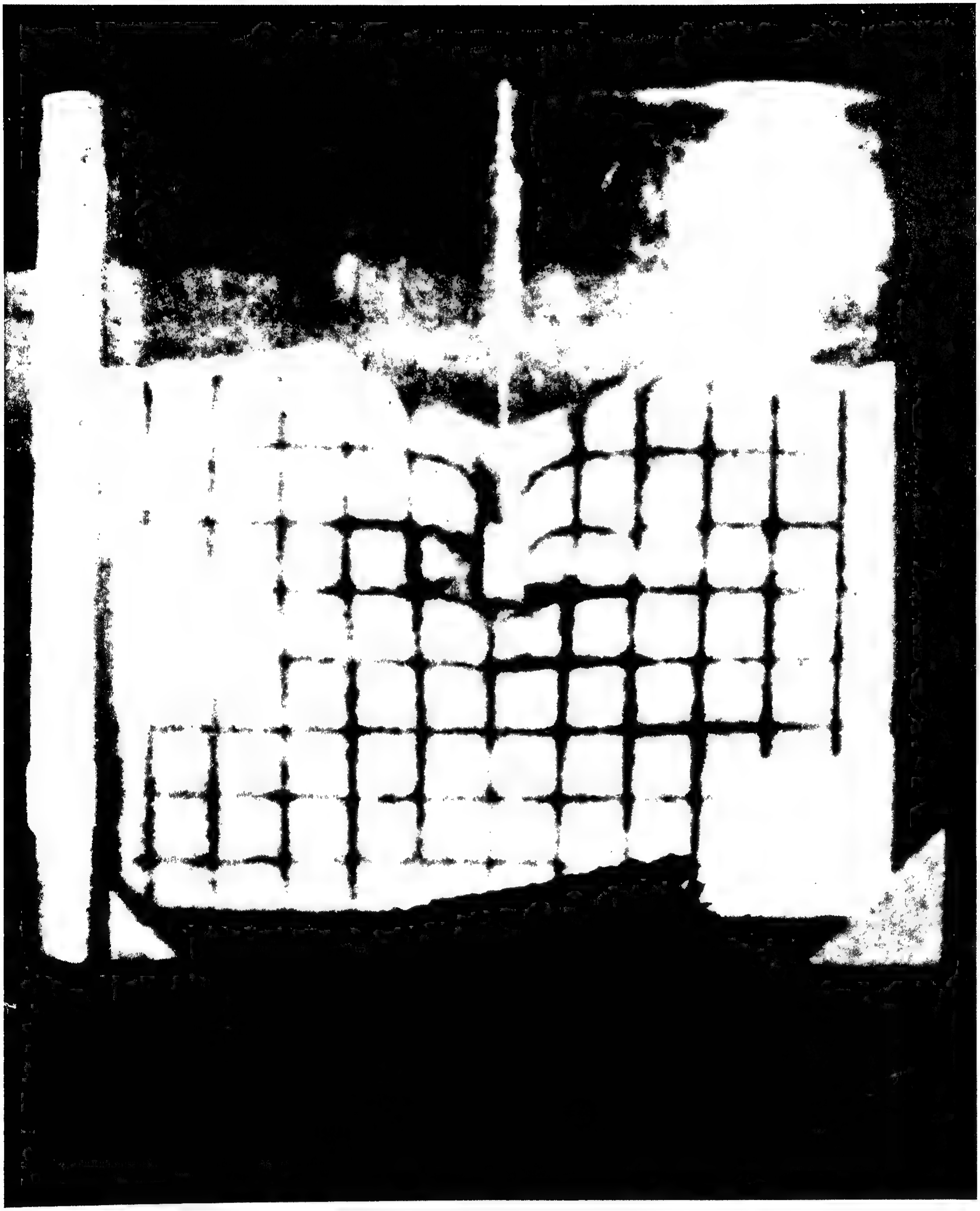

Figure 3. Preliminary Laboratory Test of Penetration Into Sand

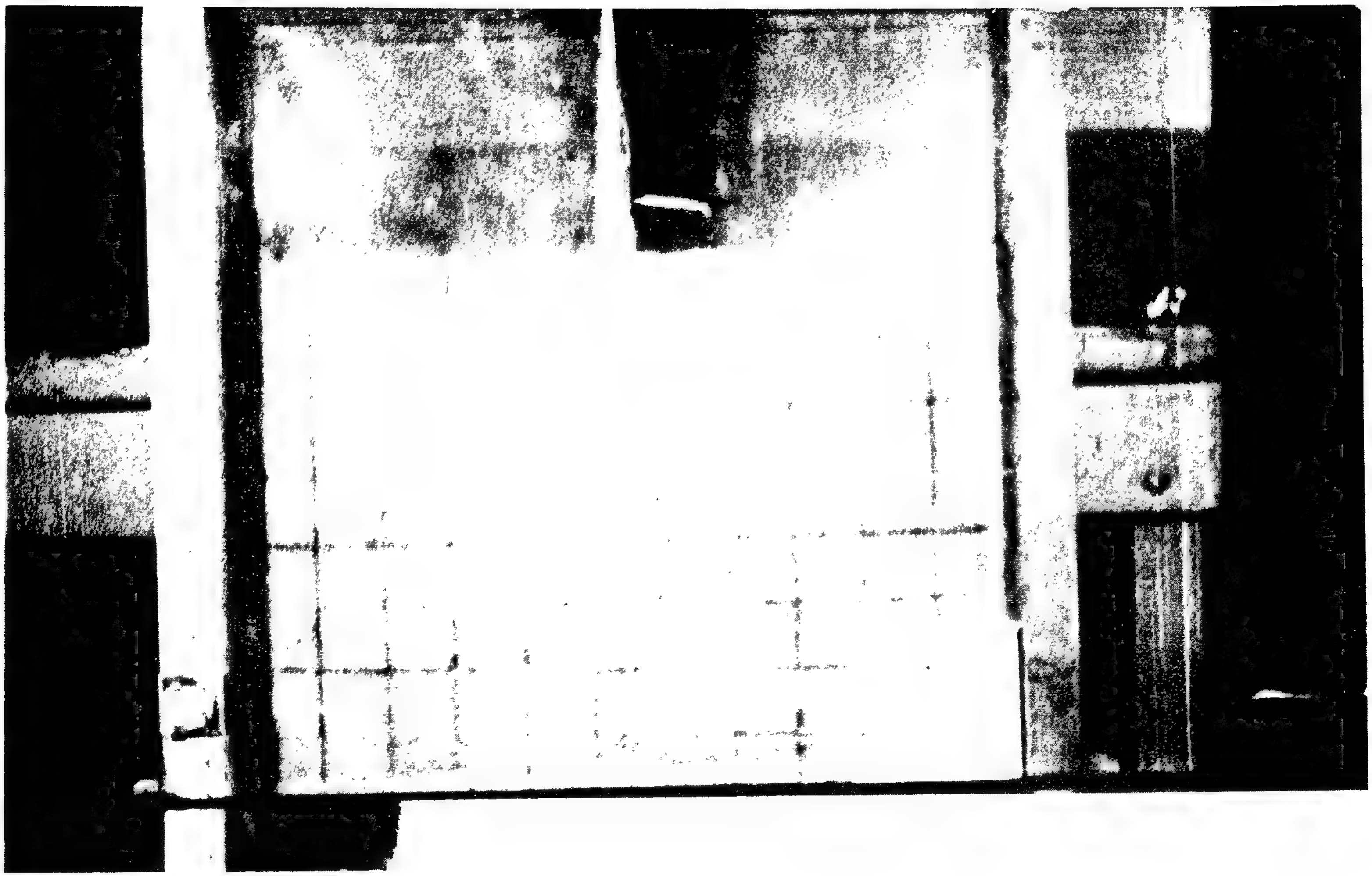

Figure 4. Preliminary Laboratory Test of Penetration Into Bulked Silt

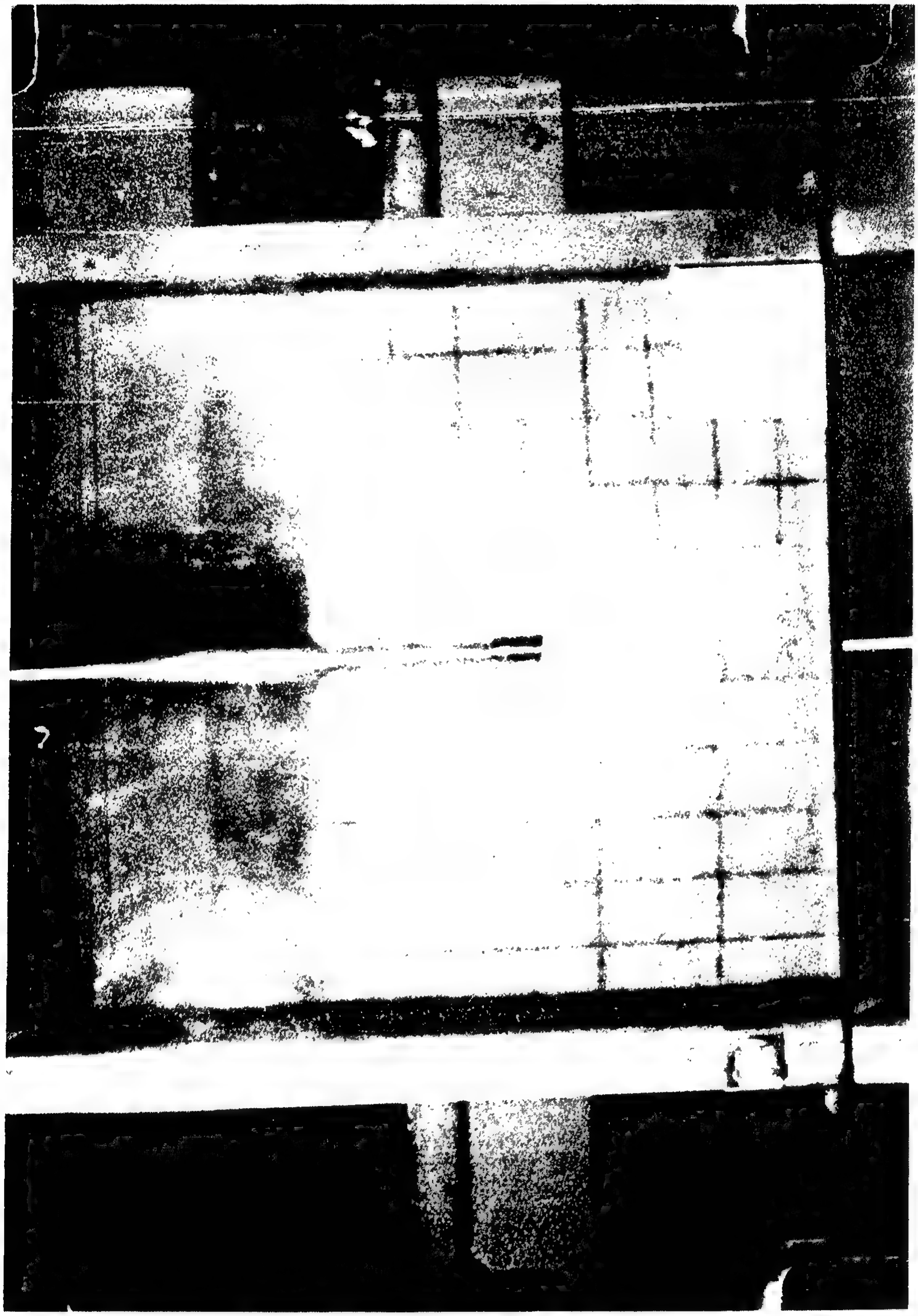

Figure 5. Preliminary Laboratory Test of Penetration Into Bulked Silt

Figure 6. Preliminary Laboratory Test of Penetration Into Bulked Silt

Figure 7. Preliminary Laboratory Test of Penetration Into Water

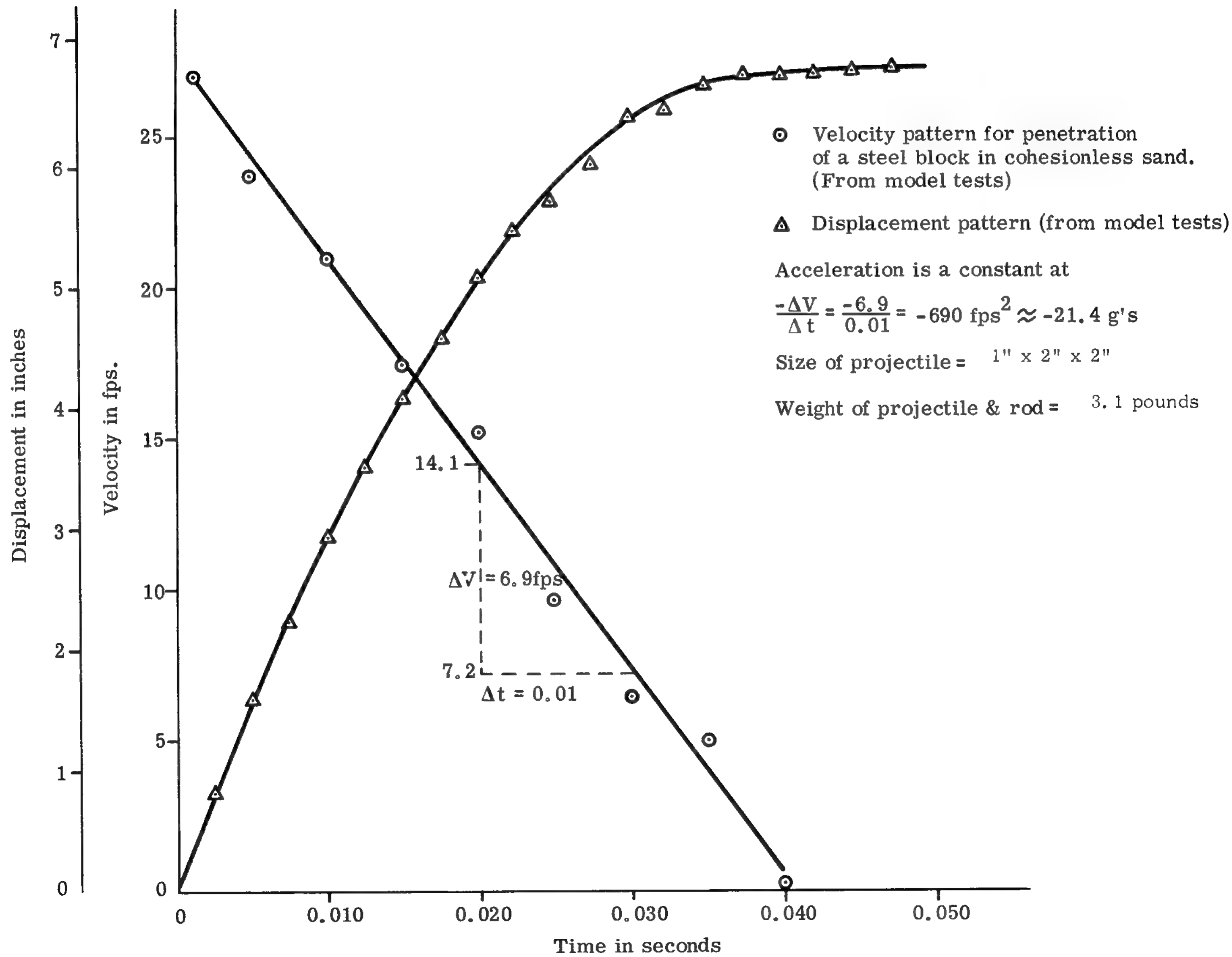

Figure 8. Vertical Penetration of Steel Block into Cohesionless Soil

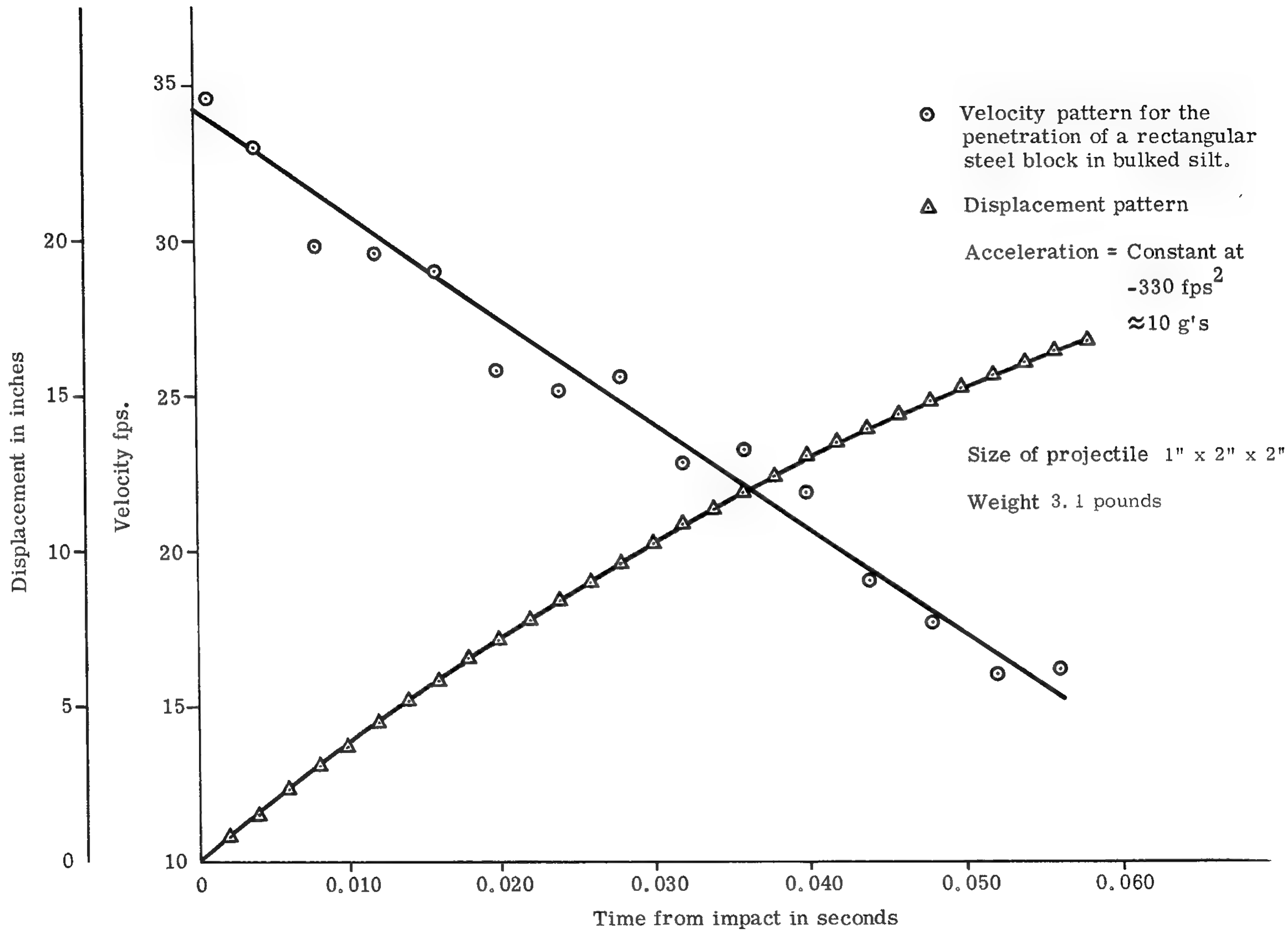

Figure 9. Vertical Penetration of Steel Block into Bulked Silt

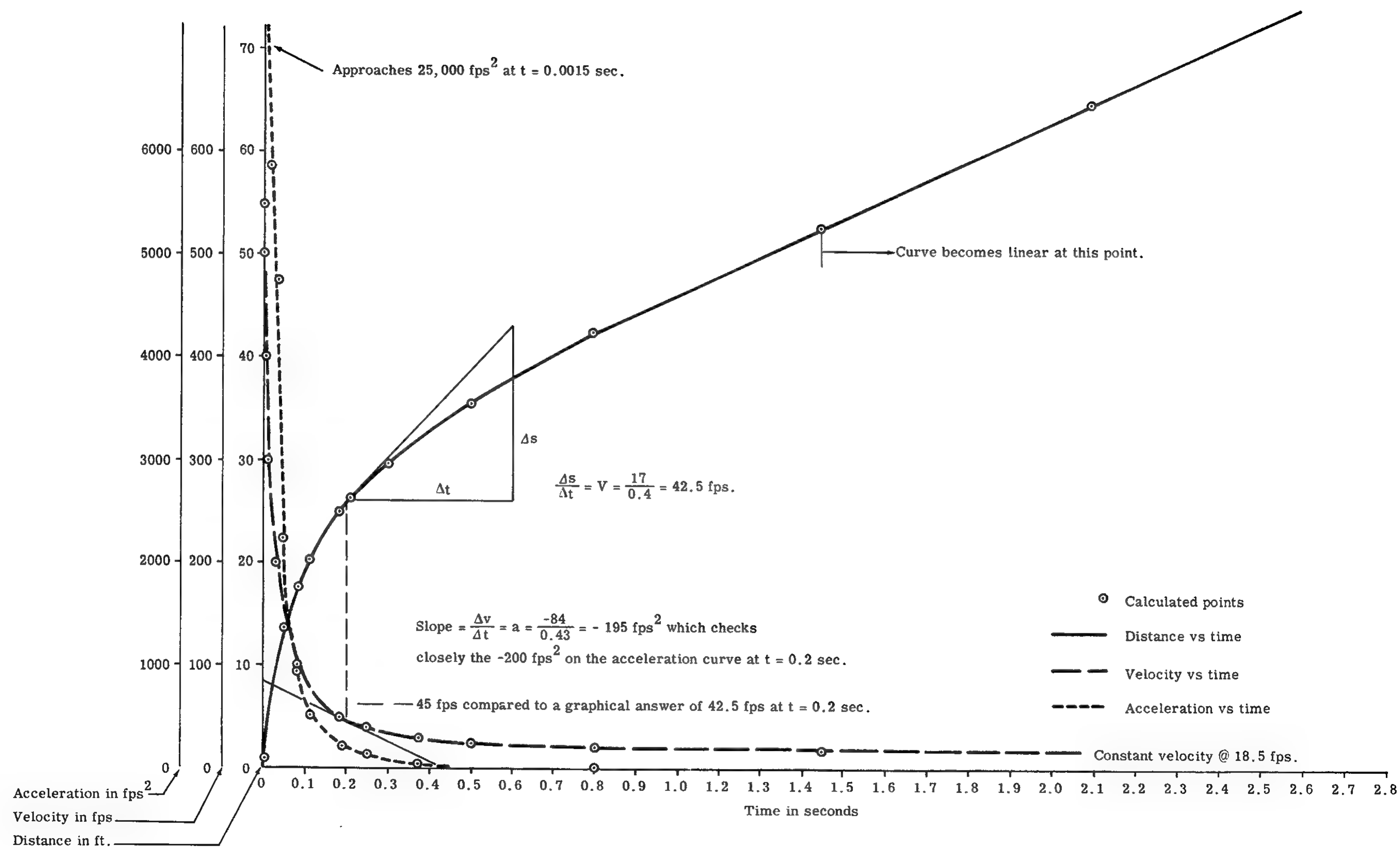

Figure 10. Theoretical Displacement - Velocity - Acceleration of Water Entry of Snap 10A Core Vessel

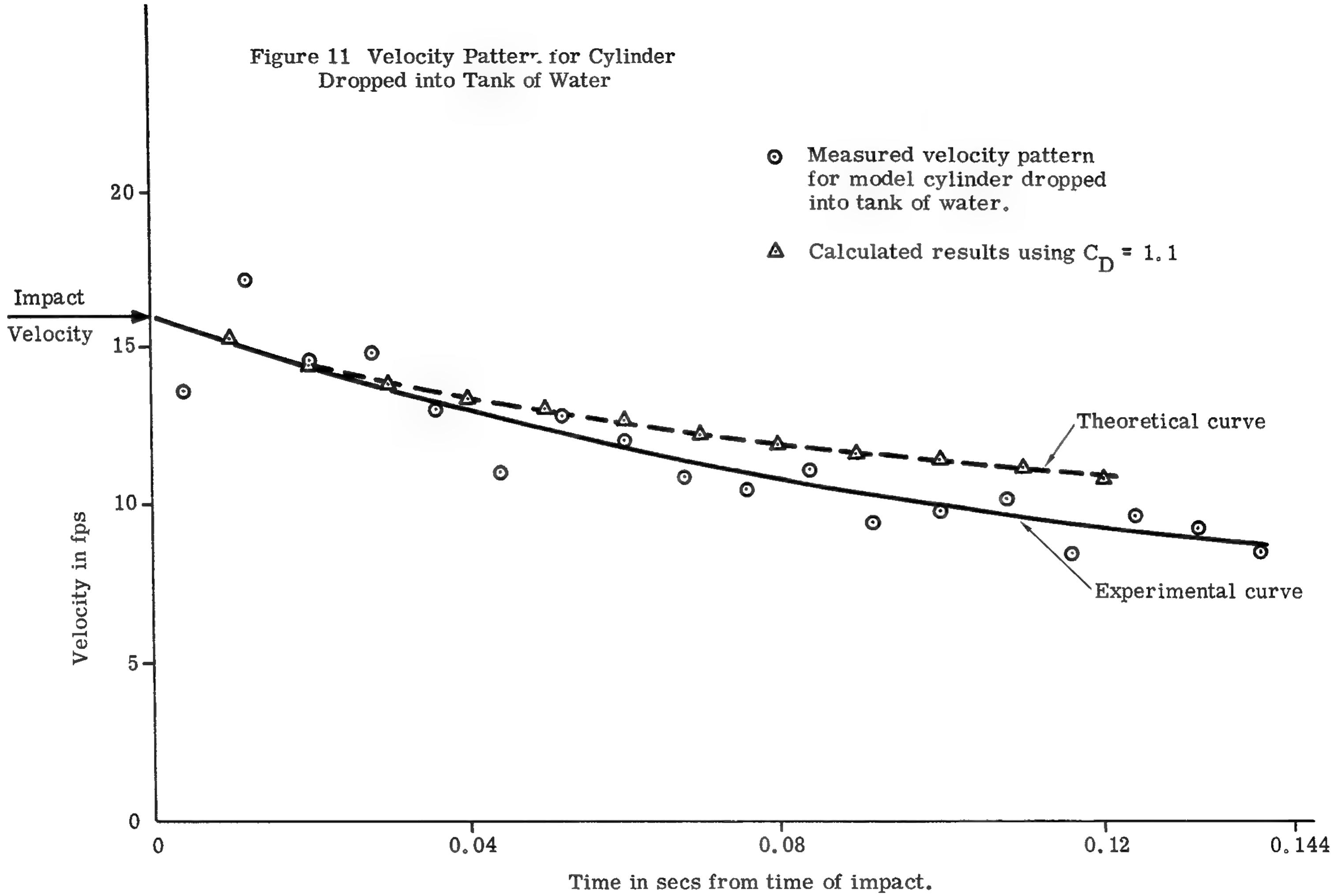

Figure 11. Velocity Pattern for Cylinder Dropped into Tank of Water

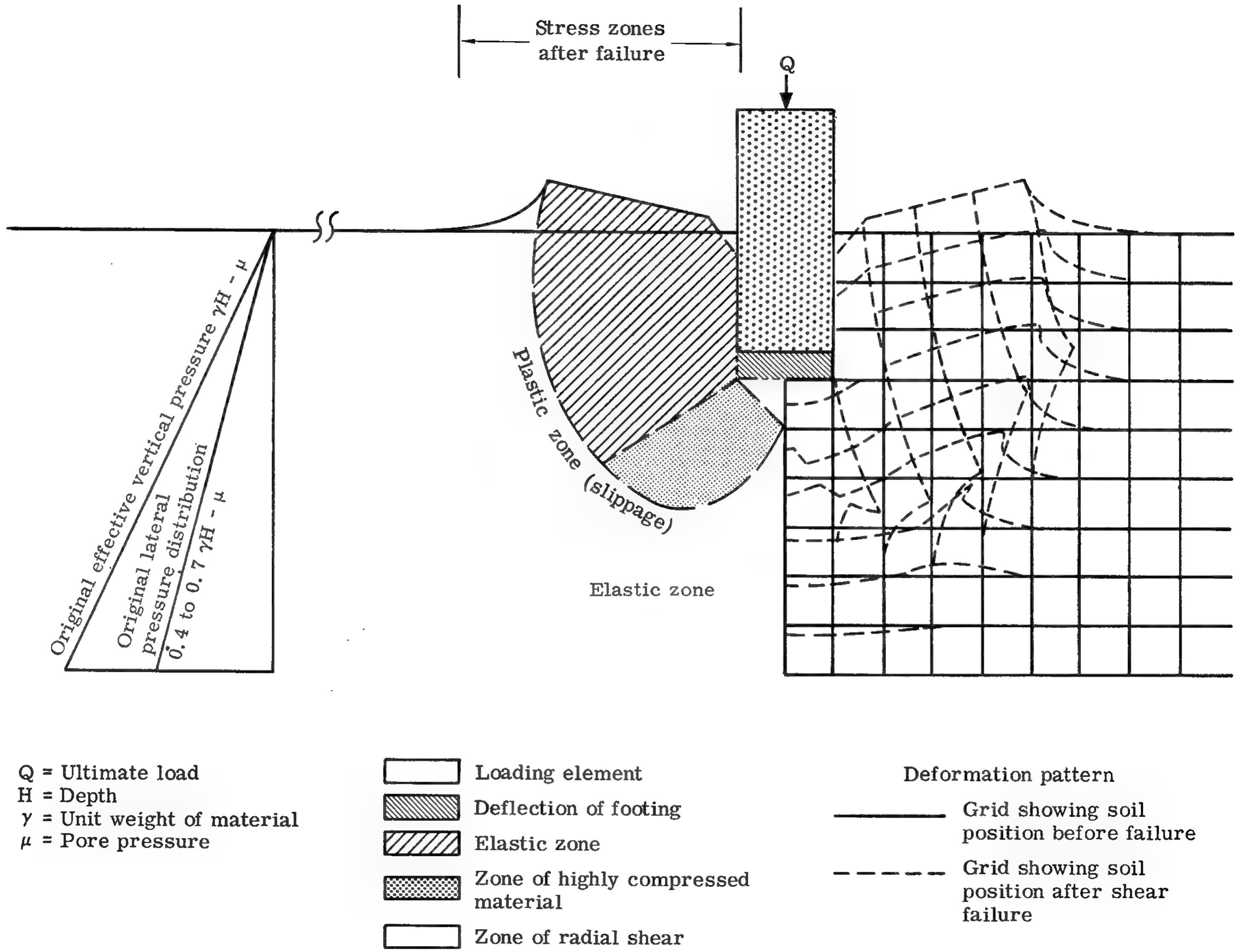

Figure 12. Cross-Section Illustrating Prandtl Plastic-Equilibrium Theory for Footing Bearing Capacity

Activity	FY 1964 S	O	N	D	J	F	M	A	M	J	FY 1965 J	A	S	O	N	D	J	F	M	A	M	J
1. Library Research	X	X	X																			
2. Liaison Visits with Other Laboratories	X																					
3. Analytical Analysis of Model Tests	X	X	X	X	X	X	X	X	X	X												
4. Laboratory Model Tests			X	X	X	X	X	X	X	X												
5. Laboratory Material Tests						X	X	X	X	X	X	X	X	X	X	X	X					
6. Full-Scale Vertical Field Tests									X	X	X	X	X	X	X	X						
7. Preliminary Full-Scale Horizontal Tests													X	X	X	X	X	X				
8. Final Full-Scale Horizontal Tests																			X	X		
9. Final Target Specification and Report																					X	X

Figure 13. Earth Target Simulation Study - Proposed Schedule

APPENDIX A

Variables controlling penetration:

Symbol	Dimensions	Description
d	L	penetration
D	L	size of projectile
ρ	FT^2L^{-4}	density of projectile
V	LT^{-1}	velocity of projectile
a	LT^{-2}	deceleration of projectile in soil
γ	FL^{-3}	unit weight of soil
c	FL^{-2}	cohesion of soil
C	FL^{-2}	compressibility of soil
ϕ	none	angle of internal friction

The assumption is made that the penetration of a projectile impacting the earth's surface is dependent on the variables listed above so that

$$d = f(D, \rho, V, a, \gamma, c, C, \phi).$$

This equation states that the dependent variable d is a function of the eight independent variables as shown. Thus, the problem is one dealing with nine variables.

In order to reduce the number of variables involved, the Buckingham Pi Theorem is applied. This procedure reduces the problem from one dealing with nine dimensional variables to one dealing with six dimensionless variables. Since the nine dimensional variables involve three dimensions, the dimensional variables can be reduced from nine to 9-3 = 6 dimensionless variables such that

$$\pi_1 = f(\pi_2, \pi_3, \pi_4, \pi_5, \pi_6).$$

This equation means that the π term involving the dependent variable is some function of the five π terms which involve independent variables. An analysis of this type often helps to organize a research project more systematically and also allows an experimenter to deal with dimensionless variables so that the number of variables involved is smaller than for the dimensional case.

The Pi-Theorem operates as illustrated in the following development:

where

L = Dimension of length

F = Dimension of force

T = Dimension of time

The functional equation is:

$$d = f(D, \rho, V, a, \gamma, c, C, \phi).$$

Choose as the repeating variables D, ρ, V so that all three dimensions are included in these variables. Then the repeating variables are combined with each of the remaining variables in the following way:

$$\pi_1 = D^{X_1} \rho^{Y_1} V^{Z_1} d^1$$

where 1 is an arbitrary number selected as the exponent of d or in dimensional form:

$$L^{X_1} (FT^2L^{-4})^{Y_1} (LT^{-1})^{Z_1} L^1 = L^o F^o T^o$$

Since each π term is to be dimensionless, the sum of the exponents of each dimension must equal 0.

$$L: \; X_1 - 4Y_1 + Z_1 + 1 = 0$$

$$F: \; Y_1 = 0$$

$$T: \; 2Y_1 - Z_1 = 0 \quad \therefore \; Z_1 = 0$$

from the equation for L

$$X_1 = -1$$

and substituting the exponents into the initial equation

$$\pi_1 = \frac{d}{D} \tag{a}$$

$$\pi_2 = D^{X_2} \rho^{Y_2} V^{Z_2} \gamma^1$$

$$L^{X_2} (FT^2L^{-4})^{Y_2} (LT^{-1})^{Z_2} (FL^{-3}) = L^o F^o T^o$$

$$L: \; Z_2 - 4Y_2 + Z_2 - 3 = 0$$

$$F: \; Y_2 + 1 = 0 \quad \therefore \; Y_2 = -1$$

$$T: \; 2Y_2 - Z_2 = 0 \quad \therefore \; Z_2 = -2$$

$X_2 = 1$ from the equation for L

and

$$\pi_2 = \frac{D\gamma}{\rho V^2} \tag{b}$$

$$\pi_3 = D^{X_3} \rho^{Y_3} V^{Z_3} a^1$$

$$L^{X_3}(FT^2L^{-4})^{Y_3}(LT^{-1})^{Z_3}(LT^{-2}) = L^oF^oT^o$$

L: $X_3 - 4Y_3 + Z_3 + 1 = 0$

F: $Y_3 = 0$

T: $2Y_3 - Z_3 - 2 = 0 \quad \therefore \quad Z_3 = -2$

$X_3 = 1$ from the equation for L

and

$$\pi_3 = \frac{Da}{V^2} \tag{c}$$

$$\pi_4 = D^{X_4} \rho^{Y_4} V^{Z_4} c^1$$

$$L^{X_4}(FT^2L^{-4})^{Y_4}(LT^{-1})^{Z_4}(FL^{-2}) = L^oF^oT^o$$

L: $X_4 - 4Y_4 + Z_4 - 2 = 0$

F: $Y_4 + 1 = 0 \quad \therefore \quad Y_4 = -1$

T: $2Y_4 - Z_4 = 0 \quad \therefore \quad Z_4 = -2$

$X_4 = 0$ from the equation for L

and

$$\pi_4 = \frac{c}{\rho V^2} \tag{d}$$

Similarly,

$$\pi_5 = \frac{C}{\rho V^2} \tag{e}$$

and since ϕ is dimensionless already, ρ, V, and D will disappear and

$$\pi_6 = \phi \tag{f}$$

The functional equation is now:

$$\pi_1 = f(\pi_2, \pi_3, \pi_4, \pi_5, \pi_6)$$

or

$$\frac{d}{D} = f\left(\frac{D\gamma}{\rho V^2}, \frac{Da}{V^2}, \frac{c}{\rho V^2}, \frac{C}{\rho V^2}, \phi\right).$$

LIST OF REFERENCES

Allely, B. H., and Newland, P. L., Discussion "On the Yielding of Soils," Geotechnique 9:2:71.

Allen, W. A., and Morrison, H. L., Projectile Energy Expended in Comminution of the Target, Naval Ordnance Test Station, China Lake, Calif., July 26, 1955, Naval Report 4885.

Allen, William A., Mayfield, Earle B., and Morrison, Harvey L., The Dynamics of Projectile Penetration in Sand, NAVORD 4980 (Uncl.), U.S. Naval Ordnance Test Station, China Lake, Calif., Dec. 7, 1955.

Allen, William A., Mayfield, Earle B., and Morrison, Harvey L., "Projectile Penetrating Sand," Journal of Applied Physics, Vol. 28, No. 3, March 1957, pp. 370-376.

ASTM, Symposium on Soil Dynamics, Special Tech. Bulletin 305, Jan. 1962. Papers before ASTM annual meeting, June 25-30, 1961.

ASTM, Symposium in ASTM Spec. Tech. Pub. No. 156. (Contains an extensive bibliography of literature prepared by subcommittee R-9.) 1953.

Barkan, D. D., Dynamics of Bases and Foundations, McGraw-Hill, New York, 1962.

Beth, R. A., Curtis, C. W., Emrich, R. J., and Others, Ordnance and Terminal Ballistics, Volume 7, A Compilation of Informal Reports Submitted in Advance of Formal Reports, Jan. 15, 1945, (CDI), Princeton University, New Jersey, Duke University, Durham, N. C., OSRD, NRDC, Div. 2, Washington, D. C., Feb. 15, 1945.

Beth, R. A., and Schaecer, E. J., An Electromagnetic Method for Measuring Projectile Velocity During Penetration, NDRC-A-329 (Uncl.).

Bingham and Cook, Plastic Flow, Gov't Printing Office, Washington, D. C.

Birkoff, Garrett, Transient Cavities in Air-Water Entry, ATI 151443, NAVORD Report 1490, Armed Services Technical Information Agency, Arlington Hall Station, Arlington 12, Virginia, Jan. 1951.

Bjerrum, L., Simons, N., and Torblaa, I., "The Effect of Time on the Shear Strength of a Soft Marine Clay," Proc. European Earth Pressure Conf., p. 148, 1958.

Bowden, F. P., and Brunton, J. H., "Deformation of Solids by Liquid Impact at Supersonic Speeds," Roy. Soc. Proc. Sci. A. v 263, n 1315, Oct. 10, 1961, pp. 433-50.

Bullen, K. E., An Introduction to the Theory of Seismology, Cambridge University Press, 1947.

Casagrande, A., and Shannon, W. L., "Stress Deformation and Strength Characteristics of Soils Under Dynamic Loads," Trans. ASCE, Vol. CXIV, pp. 755-773, 1949.

Casagrande, A., and Shannon, W. L., Research on Stress Deformation and Strength Characteristics of Soils and Soft Rocks Under Transient Loadings, Harvard University Soil Mech. Series No. 31, 1948.

Casagrande, A., and Wilson, S. D., "Effect of Rate of Loading on the Strength of Clays and Shales at Constant Water Content," Geotechnique, Vol. 2, p. 251, 1951. Also Harvard Soil Mechanics Series No. 39.

Caudle, W. N., and Pope, A. Y., Project Trump: Progress Report No. 1., SCTM 56-62(71), SRD, Sandia Corp., April 1962.

Chisnell, R. F., "Normal Motion of Shock Waves Through Nonuniform, One-Dimensional Medium," Roy. Soc. Proc. v 232, n 1190, Nov. 8, 1955, pp. 350-370.

Chou, P. Y., Water Entry and Underwater Ballistics of Projectiles, C.I.T., Pasadena, Calif., 1946, OSRD Report No. 2551, Navy Dept., Office of Scientific Research and Development.

Clough, Ray, "Earthquake Loads on Dams," ASCE Trans., 1958.

Coleman, J. D., "Suction and the Yield and Failure Surface for Soil in Principal Effective Stress Space," Correspondence Geotechnique, 10 (4), 1960, p. 181.

Cooper, E. P., Theory of Water Entry of Missiles with Flat Noses, NAVORD Report 1154-1949, U.S. Naval Ordnance Test Station, Inyokern, Calif.

Cox, A. D., "Axially Symmetric Plastic Deformation in Soils," Int. J. Mech. Sciences, v 4, Sept.-Oct. 1962, pp. 371-80.

Cox, A. D., Easton, G., and Hopkins, H., "Axially Symmetric Plastic Deformation in Soils," Royal Society of London-Phil. Trans., Series A., Vol. 254, n 1036, Aug. 24, 1961, pp. 1-45.

Crawford, C. B., The Influence of Rate of Strain on Effective Stresses in a Sensitive Clay, ASTM Spec. Tech. Pub. No. 254, pp. 36-61, 1959.

Cunny, R. W., and Sloan, R. C., Discussion "of Static and Dynamic Behavior of Small Footings," by McKee and Selig (Dec. 1961), ASCE J. Soil Mech. and Foundations Div., Aug. 1962, v 88 sm 4 pt. 1.

Dally, J. W., Experimental Means for Determining Transient Stress and Strain, Ph.D. Thesis, Illinois Institute of Technology, June 1958.

Davids, N. (Editor), International Symposium on Stress Wave Propagation in Materials, Interscience Publishers, Inc., New York, 1960.

Delaney and Sorensen, Low Speed Drag of Cylinders of Various Shapes, NACA T. Note 3038, 1953.

Dergarbedian, Paul, Compressibility Effects During Water Entry, NOTS 1159, NAVORD Report 3523, Underwater Ordnance Dept., China Lake, California, June 1955.

Drucker, D. C., "Coulomb Friction, Plasticity and Limit Loads," J. of Applied Mechanics, Vol. 21, 1954, pp. 33-43.

Drucker, D. C., "Limit Analysis of Two and Three Dimensional Soil Mechanics Problems," Journal of the Mechanics and Physics of Solids, Vol. 1, 1953, pp. 217-226.

Drucker, D. C., "A More Fundamental Approach to Plastic Stress-Strain Relations," Proc., First U.S. National Congress of Applied Mech., Chicago, Ill., 1951, p. 487.

Drucker, D. C., and Prager, W., "Soil Mechanics and Plastic Analysis or Limit Design," Quarterly of Applied Math., Vol. 10, 1952, pp. 157-165.

Drucker, D. C. and Shield, R. T., "Application of Limit Analysis to Punch Indentation Problems," ASME Trans. (J. Applied Mech.), v 20, n 4, Dec. 1953, pp. 453-460.

Durelli, W. F., and Riley, W. F., Application of Moire Methods to the Determinations of Transient Stress and Strain Distributions, ASME (Applied Mechanics Div.), Nov. 23, 1963 #61-WA-10.

Durvall, G. E., Shock Waves in Solids, International Science and Technology, April 1963.

Ewing, W. A., Jardetzky, W. S., and Press, F., Elastic Waves in Layered Media, McGraw-Hill, New York, 1957.

Fisher, Walter E., Experimental Studies of Dynamically Loaded Footings on Sand, Tech. report to U.S. Army Engineer Waterways Experiment Station, (University of Illinois), July 1962.

Flaherty, R. M., "Basis of Photoelasticity," American Soc. Naval Engrs. J., v 73 n 1, Feb. 1961.

Flanders, Phyllis L., and Sauer, Fred M., A Glossary of Geoplosics: The Systematic Study of Explosion Effects in the Earth, AFSWC-TN-60-20 (Uncl.), Stanford Research Institute, July 31, 1960.

Freeborn, R. I. C., Penetration of Projectiles into Soil, (Secret), Ministry of Supply, Dept. of Scientific and Industrial Research, London, 1947.

French, A. P., Penetration of Ballistics Projectiles (Final Report), SCU-AD-145994 (Uncl.), University of South Carolina Dept. of Physics, 1957.

Geniev, G. A., "Problems of Dynamics of Soil Media," (in Russian), Academy of Construction and Architecture, USSR, Moscow, 1958.

Gibson, R. E., and Henkel, D. J., "Influence of Duration of Tests on 'Drained' Strength," Geotechnique, Vol. 4, p. 6, 1954.

LIST OF REFERENCES (continued)

Goldsmith, Werner, Impact, E. Arnold, 1960.

Goldstein, G., Modern Developments in Fluid Dynamics, Oxford at the Clarendon Press, London, 1938.

Goodier, Jahsman, and Ripperger, "Experimental Surface-wave Method for Recording Force Time Curves in Elastic Impacts," ASME Trans., J. Applied Mech., v 26, ser E, n 1, Mar. 1959, pp. 3-7.

Green, C. B., Final Summary Report on Project Elsie (SRD), Dept. of the Navy, Bureau of Ordnance, Washington, D. C., BUO-REM-3, April 1, 1952.

Gumenskii, B. M., and Komarov, N. S., Soil Drilling by Vibration, Russian, English trans. by Consultants Bureau, 227 West 17th St., New York 11, N. Y.

Harri, J. G., A Modification of the Petry Soil Penetration Equation, SCTM 250-62(71) (CDI), Sandia Corp., December 1962.

Harri, John, Soil Penetration Theory, SCTM 225-62(71) (CFRD), Sandia Corp., December 1962.

Hendron, A. J., The Energy Absorption Capacity of Granular Materials in One-Dimensional Compression, AFSWC-TDR-62-91, Final Report, Jan. 1963.

Henkel, David J., Discussion "on the Yielding of Soils," Geotechnique, Vol. 8, p. 134, 1958.

Henkel, D. J., and Skempton, A. W., "A Landslide at Jackfield, Shropshire, in a Heavily Over-consolidated Clay," Geotechnique, Vol. V, pp. 131-137, 1955.

Hoerner, Sighard F., Aerodynamic Drag, Midland Park, N. J., 1951, 259 pp.

Hoerner, Sighard F., Fluid-Dynamic Drag, Midland Park, N. J., 1958, Chapter on pressure drag. Chap. 3, 28 pp.

Housel, W. S., "Dynamic and Static Resistance of Cohesive Soil: 1946-1958," Paper presented to 62nd Annual Meeting of ASTM, 1959, STP 254, pp. 4-35.

Hurley, P. M., and others, Photographic Studies in Underwater Ballistics, ATI 35541, California Institute of Technology, OSRD $OEM_{SR}418$, Nov. 15, 1945.

Jenike, A. W., and Sheild, R. T., "On the Plastic Flow of Coulomb Solids Beyond Original Failure," Trans. of ASME Series E, J. of Mechanics, 1959, pp. 599-602.

Jones, R., "In Situ Measurements of Dynamic Props. of Soil by Vibration Methods," Geotechnique, v 8, n 1, Mar. 1958, pp. 1-21.

Kaufmann, Walter, Fluid Mechanics, McGraw-Hill, N. Y., 1963, 432 pp., p. 195, "Surface Waters."

Kerr, William C., "The Dynamic Passive Pressure Problem for Sand," M. I. T. Research Report R63-30, May 1963.

Khachaturian, Narbey, "Survey of Literature in Connection with the Dynamic Bearing Capacity of Soils," DASA 1161, AD-232 102, Oct. 1959. (Found in Vol. 37, no. 4, p. s14 of the Govt. research reports.)

Khalifa, Mohamed K., Model Studies of Pile Foundations, Civil Engr. Research Labs., Columbia Univ., 1940-#7.

Kolsky, H., Stress Waves in Solids.

Kondner, R. L., "Vibratory Simple Shear in Clay," paper presented to 40th Annual Meeting of Highway Research Board, 1961.

Korkegi, Robert H., Pressure Distribution on a Sphere Entering Water Based on Various Linearized-Theoretic Models, Underwater Ballistics Division, Underwater Ordnance Dept., Local Project 329, Sept. 1950, N.O.T.S., Pasadena Annex, Inyokern, California.

Langhaar, Henry L., Dimensional Analysis and Theory of Modeling, Wiley.

Livingston, Clifton W., Bomb Penetration Tests, Fort Churchill, Canada, SIPRE-TR-71, U.S. Army Snow, Ice and Permafrost Research Establishment, Corps of Engineers, Wilmette, Illinois, Sept. 1960.

Livingston, Clifton W., Bomb Penetration Tests, Fort Churchill, Canada, Technical Report 71, U.S. Army Snow, Ice and Permafrost Research Establishment, September 1960.

Livingston, C. W., and Smith, F. L., Selected Sections From: Bomb Penetration Project, CSM-280-51-4 (Uncl.), Colorado School of Mines Research Foundation, Inc., June 15, 1951.

Longman, Grunzweig, and Petch, "Calculations and Measurements on Wedge Indentation," J. Mech. and Physics of Solids, v 2, n 2, Jan. 1954, pp. 81-6.

Macelwane, J. B., Introduction to Theoretical Seismology, Part 1, Geodynamics, Wiley, New York, 1936.

Maloney, F. J., An Evaluation of the TX-8' (Externally Carried Mark 8 Weapon), SC-1936 (TR), (CRD), Sandia Corp., July 1, 1951.

May, Albert, "Vertical Entry of Missiles into Water," Journal of Applied Physics, Vol. 23, No. 12, Dec. 1952, pp. 1362-1372.

Mayfield, Earle B., Morrison, Harvey L., and Allen, William A., Projectile Penetration into Sand, NOTS-Q-4 (Uncl.), Michelson Lab., U.S. Naval Ordnance Test Station, China Lake, Calif.

McKee, K. E., "Foundations for Protective Structures," Proceedings of the 29th Shock and Vibration Symposium, 1960.

McKee, K. E., "Design and Analysis of Foundations for Protective Structures," Proceedings of the Fourteenth Panel Meeting on Blast Effects on Building and Structures and Protective Construction, Oct. 1958.

McKee, K. E., "The Dynamic Behavior of Spread Footings," Ph.D. Thesis, Illinois Institute of Technology, 1962.

McKee, K. E., Design and Analysis of Foundations for Protective Structures, AFSWC-TR 59 56, Oct. 1959.

McKee, K. E., Design and Analysis of Foundations for Protective Structures, Interim Report, AFSWC-TN 60 36, Sept. 1960.

McKee, K. E., Design and Analysis of Foundations for Protective Structures, Second Interim Report, AFSWC-TDR 61 14, May 1961.

McKee, K. E., and Shenkman, S., Design and Analysis of Foundations for Protective Structures, Final Report, AFSWC-TDR 62 9, Jan. 1962.

Meyerhoff, G. G., "Compaction of Sands and Bearing Capacity of Piles," 1961 Trans., p. 1292.

Meyerhoff, G. G., "Penetration Tests and Bearing Capacity of Cohesionless Soils," ASCE Proc., v 82 (J. Soil Mech. and Found. Div.) n, sml, Jan. 1956, paper 866, 19pp.

Milne-Thompson, L. M., Theoretical Hydrodynamics, MacMillan Co. 1938, pp. 552, London, Chap. 17.

Mogami, T., and Kubo, K., "The Behavior of Soil During Vibration," Proc. 3rd Int. Conf. on Soil Mechanics and Foundation Eng., Vol. 1, pp. 152-155, 1953.

Morrison, H. L., Effect of Projectile Nose Shape on the Depth of Penetration into a Brittle Porous Medium, NOTS 1084, NAVORD 3473, AD 62496 (Uncl.), U.S. Naval Ordnance Test Station, Inyokern, China Lake, Calif., Apr. 5, 1955.

Morrison, H. L., Projectile Energy Expended in Comminution of the Target, NOTS 1181, NAVORD 4885 (Uncl.), U.S. Naval Ordnance Test Station, Inyokern, China Lake, Calif., July 26, 1955.

Morrison, H. L., and Shyburn, W. M., Preliminary Law for Projectile Penetration into Porous Media, NAVORD 2098, U. S. Naval Ordnance Test Station, Inyokern, China Lake, California, April 27, 1954.

Murphy, G. M., Similitude in Engineering, Ronald Press Co., New York, 1950 (Eng. Soc. Lib. N. Y.).

Nara, Harry R., Clark, Samuel K., and Rudnick, Alfred, Project Doan Brook - Model Studies of Aerial Missiles Fired into Sand Surfaces, (SDI), Case Institute of Technology, University Circle, Cleveland 6, Ohio, October 4, 1954.

Nara, Harry R., and Denington, Robert J., Project Doan Brook-Richochet and Penetration of Steel Spheres in Clay and Sand Soils, (SDI), Case Institute of Technology, University Circle, Cleveland 6, Ohio, October 18, 1954.

Nara, Harry R., and Rudnick, Alfred, Project Doan Brook-Effects of Certain Parameters on the Penetration of Missiles in Sand, (SDI), Case Institute of Technology, University Circle, Cleveland 6, Ohio, October 11, 1954.

Nauman, Tests on Cylindrical Shapes in Axial Flow Cylinders - ZWB, Tech. Berichte 1943, p. 278.

Nikolaev, B. A., Pile Driving by Electroosmosis, Russian, English translation by Consultants Bureau, 227 West 17th St., New York 11, New York.

Osterman, J., "Theoretical Study of Failure Conditions in Saturated Soils," Sweden, Geotechnical Inst., Proc. 20, 1962, p. 34.

Ostrovskii, A. P., Deep-Hole Drilling with Explosives, Russian, English translation by Consultants Bureau, 227 West 17th St., New York, 11, N. Y.

Parkin, B. R., "Impact Waves in Sands: Theory Compared With Experiments on Sand Columns," Proc. ASCE, Vol. 87, No. sm3, pp. 1-32. (See discussion by Whitman in Vol. 88, No. sml, pp. 49-58, 1961.)

Parry, R. H. G., Discussion "on the Yielding of Soils," Geotechnique, 8:4:183.

Peck, Hanson, and Thornburn, Foundation Engineering, Wiley, New York, 1953.

Peltier, M. R., "Experimental Investigations on the Intrinsic Rupture Curve of Cohesionless Soils," Proc. of the Fourth International Conference on Soil Mech. and Foundation Engr., London, 1957 (in French), pp. 179-182.

Penetration of Projectiles into Concrete, Rock, and Soil, Textbook of Air Armament, Part 1 (Theory of Damage and Weapon Performance), Chapter 3, (Secret), Admiralty; Air Ministry, England, C. B. 04512A, Chapter 3, February 1949.

Poorooshasb, H. B., and Roscoe, K. H., "The Correlation of the Results of Shear Tests With Varying Degrees of Dilation," Proc. 5th Int. Conf. Soil Mech., 1961.

Rao, N. S. Govinda, and Nagaraj, C. N., "Soil Dynamics," Irrig. and Power Journ. (India), v 12, n 3, July 1955, pp. 450-9, Biblio.

Recovery of the SP-8 (TX-8) (38M), (Mark 8) Bomb to Obtain Penetration and Bomb Case Damage Data, Second Report (Final), (Secret), Ordnance Corps, Aberdeen Bombing Mission Development and Proof Services, Edwards Air Force Base, Edwards, Calif., July 2, 1952.

Reichardt, H., The Laws of Cavitation Bubbles at Axially Symmetrical Bodies in a Flow, A.T.I. 44252, Armed Services Technical Information Agency, Arlington Hall Station, Arlington 12, Virginia, Mar. 1960.

Richardson, "Effects of Rate of Strain on Stress Strain Behavior of Sat. Soil," M.I.T. Publication (Soils Engr. Division).

Richter, C. F., Elementary Seismology, W. H. Freeman and Company, San Francisco, 1960.

Riley, W. F., Daniel, I. M., and Durelli, A. J., Stress Wave Phenomena in Semi-Solids, Armour Research Foundation of Illinois Institute of Technology, July 1961, AFSWC-TR-61-25.

Rinehart, J., Fractures Caused by Explosives and Impacts, Quarterly of Colorado School of Mines, Vol. 55, No. 4.

Roscoe, K. H., Schofield, A. N., and Wroth, C. P., "On the Yielding of Soils," Geotechnique, 8:1:22.

Roscoe, K. H., and Poorooshasb, H. B., "A Theoretical and Experimental Study of Strains in Triaxial Tests on Normally Consolidated Clays," Geotechnique, 13:1:12-38, 1963.

Roscoe, M. K. H., Discussion on "Soil Properties and Their Measurements," Proc. 5th Int. Conf. Soil Mech. and Found. Engr., 1961.

Roscoe, Schofield, and Thurairajah, "Yielding of Clays in States Wetter Than Critical," Geotechnique, Vol. XIII, n 3, Sept. 1963.

Roscoe, Schofield, and Wroth, Correspondence "On the Yielding of Soils," Geotechnique, 9:2:72-83, 1959. (A reply to correspondence by Parry, Smoltczyk, and Henkel.)

Roscoe, Schofield, and Wroth, "A Critical Appreciation of Test Data for Selecting a Yield Criterion for Soils," Symposium on Laboratory Shear Testing of Soils, Ottawa, Canada, Sept. 1963.

Schofield, A. N., "The Development of Lateral Force During the Displacement of Sand by the Vertical Face of a Rotating Model Foundation," Ph.D. Thesis, Cambridge University, 1959.

Scott, R. F., Principles of Soil Mechanics, Addison-Wesley Publishing Co., Inc., 1963.

Sedov, L. I., Similarity and Dimensional Methods in Mechanics, Academic Press, New York and London, 1959.

Seed, H. B., "Soil Strength During Earthquakes," Proc. 2nd World Conf. on Earthquake Eng., Vol. 1, pp. 183-195, 1961.

Seed, H. B., and Lundgren, R., "Investigation of Transient Loading on the Strength and Deformation Characteristics of Saturated Sands," Proc. ASTM, v 54.

Selig, E. T., "Technique for Observing Structure Soil Interaction," Materials Research and Standards, v 1, n 9, Sept. 1961, pp. 717-19.

Selig, E. T., "Review of Literature Concerning Behavior on Foundations Under Dynamic Loads," Symposium on Foundation Engineering, Indian Institute of Science, Jan. 1961.

Selig, E. T., "Response of Foundations to Dynamic Loads," Thesis, Illinois Institute of Technology, Jan. 1960.

Selig, E. T., and McKee, K. E., "Static and Dynamic Behavior of Small Footings," J. of Soil Mech. and Foundation Div., ASCE v 87, No. sm6, Dec. 1961.

Shield, R. T., "Mixed Boundary Value Problems in Soil Mechanics," Quarterly of Applied Math, Vol. 11, 1953, pp. 61-75.

Shield, R. T., "Stress and Velocity Fields in Soil Mechanics," J. Math Phys., Vol. 33, 1954, pp. 144-156.

Shenkman, S., and McKee, K. E., "Bearing Capacity of Dynamically Loaded Footings," Symposium on Soil Dynamics, ASTM, June 1961.

Slade, J. J., Jr., "A Discontinuous Model for the Problems of Soil Dynamics," Symposium on Dynamic Testing of Soils, ASTM, pp. 69-76 (1953). (Issued as separate pub. ASTM STP No. 232.)

Slichter, L. B., Modeling of Water Entry of Bombs and Projectiles, Report for Office of Scientific Research and Development from California Inst. of Technology.

Smoltczyk, Hans-U, Discussion "On the Yielding of Soils," Geotechnique 8:4:186.

Sobotka, Z., "Slip Lines and Slip Surfaces in Theory of Plasticity and Soil Mechanics," Applied Mechanics Rev., Vol. 14, n 10, Oct. 1961, pp. 753-759.

Staud, A. H., and Epley, R. H., LCdr, USN, Final Report, Hard-Impact Tests of Test Missile EX-730, NAVORD-NOTS-653, (SRD), U.S. Naval Ordnance Test Station, Inyokern, China Lake, California, June 30, 1953.

Stipe, J. G., Jr., (National Research Council), Penetration in Soils, Interim Report No. 30, Report to Chief of Engrs., U.S. Army, Committee on Fortification Design, July 1944. Also NRC-IM-30 (Uncl.), National Research Council.

Stoker, J. J., Water Waves, Interscience Publishers, Inc., New York 1957, 567 pp. (Large bibliography).

Sylwestrowuz, W., "Experimental Investigation of Behavior of Soil Under Punch Footings," J. Mech. and Physics of Solids, v 1, n 4, July 1953, pp. 258-64.

Terminal Report for Project Ferret, Prepared by the Ferret Committee, NOLR-1191, (SRD), U.S. Naval Ordnance Lab., White Oak, Md., Mar. 1, 1954.

Terzaghi, K., Theoretical Soil Mechanics, John Wiley, New York.

Thompson, W. J., "Some Deformation Characteristics of Cambridge Cault Clay," Ph.D. Thesis, Cambridge University, 1962.

Thurairajah, A., "Some Properties of Kaolin and of Sand," Ph.D. Thesis, Cambridge University.

Tillett, J. R. A., "Study of Impact Spheres on Plates," Physiol Soc. of London Proc., v 67, n 417, b pt. 9, Sept. 1954, pp. 667-88.

Torpedo Studies, Summary Technical Report of Division 6, NDRC, Vol. 21, Office of Scientific Research and Development, Washington, D. C., 1946.

Trilling, George H., Splash Contours in Water Entry III - Oblique Entry of Cylinders and Spheres, Underwater Ballistics Division, Underwater Ordnance Dept., Local Project 329, 1950, N.O.T.S., Inyokern, Calif.

Trilling, L., The Compression Shock Due to the Impact of a Body on a Plane Water Surface, Underwater Ballistics Division, Underwater Ordnance Dept., Local Project 329, Sept. 1950, N.O.T.S., Inyokern, California.

USAF (UN 53-7), "Electro Mechanical Pressure Gages for Dynamic Pressure Recording," 33(616)-266: Armour Research Foundation, December 1953.

USAF (TN 58-18), "Investigation of Wave Propagation in Semi-Solids," 29(601)-465: Armour Research Foundation, March 1958.

USAF (TN 58-25), "Initial Investigation of Wave Propagation in Large Soil Models," 29(601)-796: Armour Research Foundation, December 1958.

USAF (TN 61-6), "A Theoretical Study of Structure-Medium Interaction," 29(601)-2838: National Engineering Science Company, November 1960.

USAF (TN 61-7), "Basic Interaction Considerations--Dynamic Stress Concentrations Around Unlined Openings," In-house, Feb. 1961.

USAF (TR 60-4), "Research Studies of Stress Waves in Earth and Model Earth Media," 29(601)-1167: Armour Research Foundation, October 1959.

USAF (TR 60-55), "Research Studies on Free Field Instrumentation," 29(601)-1944: United Electrodynamics Company, December 1960.

USAF (TR 61-7), "Development of a Miniature Dynamic Pressure Gage," 29(601)-1200: Armour Research Foundation, Jan. 1961.

USAF (TR 61-32), "Static Experiments for the Study of the Interaction of Buried Structures with Ground Waves," 29(601)-2652: Armour Research Foundation, April 1961.

USAF (TR 61-48), "Preliminary Design Study for a Dynamic Soil Testing Laboratory," 29(601)-1947: Massachusetts Institute of Technology, July 1961. (Small scale footing studies, A Review of the Literature--Appendix B.)

USAF (TR 61-51), "Ground Shock Isolation of Buried Structures," 29(601)-2586: Armour Research Foundation, August 1959.

USAF (TR 61-58), "Preliminary Design Study for a Dynamic Soil Testing Laboratory," Appendixes K, L, M, and N, 29(601)-1947: Massachusetts Institute of Technology, August 1961.

USAF (TDR 61-93), "Geologic Structure Stability and Deep Protection Construction," 29(601)-2821: University of Missouri, November 1961.

USAF (TDR 62-2), "Study of the Use of Models to Simulate Dynamically Loaded Underground Structures," 29(601)-4375: Iowa State University, January 1962.

USAF (TDR 62-3), "Study of the Use of Models to Simulate Dynamically Loaded Underground Structures," 29(601)-4374: American Machine and Foundry Company, Feb. 1962.

USAF (TDR 62-30), "Theoretical Study of Structure-Medium Interaction," 29(601)-2838: National Engineering Science Company, Mar. 1962.

USAF (TDR 62-43), "Theoretical Study of Energy Distribution in a Half-Space Under Dynamic Loads," 29(601)-2832: General American Transportation Corporation, July 1962.

USAF (TDR -47), "Study of Stress Wave Interaction with Buried Structures," 29(601)-4312: Armour Research Foundation, May 1962.

USAF (TDR 62-90), "Laboratory Experiments on the Response of Soils to Shock Loadings," 29(601)-2830: Purdue University, Jan. 1963.

USAF (TDR 62-91), "Energy Absorption Capacity of Granular Materials in One-Dimensional Compression," 29(601)-4302: University of Illinois, July 1962.

USAF (TDR 62-126), "Symposium on the Equation of State for Earth Materials," In-house, Nov. 1962.

USAF (TDR 62-153), "Experiments on the Measurement of the Response of Rocks to Dynamic Loads," 29(601)-2821: Missouri School of Mines, March 1963.

USAF (TDR 63-2), "Step Load Moving with Low Subseismic Velocity on the Surface of a Half-Space of Granular Material," 29(601)-2855: Paul Weidlinger and Associates, April 1963.

USAF (TDR 63-3), "Development of a Small Soil Strain Gage," 29(601)-5343: Armour Research Foundation, March 1963.

USAF (TDR 63-6), "Study of the Collapse of Small Soil-Surrounded Tubes," 29(601)-4927: Massachusetts Institute of Technology, June 1963.

USAF (TDR 63-43), "A Theoretical Analysis of Stress Wave Interaction in a Model Soil," 29(601)-5395: National Engineering Science Company, June 1963.

USAF (TDR 63-48), "Feasibility Study of Methods for Dynamic Three-Dimensional Photoelasticity," 29(601)-5362: Armour Research Foundation, June 1963.

Von Huene, Roland, Preliminary Observations on the Penetration of Bombs into Clay, NAVORD-TRI-122, (CDI), U.S. Naval Ordnance Test Station, China Lake, California, April 29, 1955.

Wallace, W. L., "Displacement of Long Footings by Dynamic Loads," ASCE, Vol. 87, No. sm5, Oct. 1962. (See discussion by Triandafilidis in April 1962 issue.)

Waugh, J. G., and Ager, R. W., Water-Entry Whip and Deceleration of Six Full-Scale Torpedo Heads with Ogive and Plate-Ogive Heads, NAVORD Report 1308, 1951, N.O.T.S., Inyokern, China Lake, Calif.

Waugh, J. G., and Stubstad, G. W., Water Entry Cavity Modeling Part 1 - Vertical Cavities, NAVORD Report 5365, N.O.T.S., China Lake, Calif., 1956.

White, C. R., (Unfinished paper), Naval Civil Engr. Lab., Port Hueneme, California.

White, Frank G., Photographic Studies of Splash in Vertical and Oblique Water Entry of Spheres, NAVORD Report 1228, 1950, Naval Ordnance Test Station 286.

Whitman, R. V., "The Behavior of Soils Under Dynamic Loadings: Review and Preliminary Correlation of Laboratory and Field Studies," Report No. 4 by M.I.T. to Office of the Chief of Engineers, U.S. Army.

Whitman, R. V., "Testing Soils with Transient Loads," Proc. Conf. on Soils for Engineering Purposes, Mexico City, ASTM Spec. Tech. STP 232, p. 242, 1957b.

Whitman, R. V., Strength Behavior During Single Transient Loading, M.I.T., Sept. 1962.

Whitman, R. V., The Behavior of Soils Under Dynamic Loadings, Report No. 3 to Office of Chief of Engineers, U.S. Army, et al., 1954.

Whitman, R. V., "The Behavior of Soils Under Transient Loadings," Proc. 4th Int. Conf. SMFE, Vol. 1, p. 207, 1957a.

Whitman, R. V., and Healy, K. A., "Shear Strength of Sands During Rapid Loading," Proc. ASCE, Vol. 88, No. Sm2, pp. 99-132, 1962.

Whitman and Healy, "Shearing Resistance of Sands During Rapid Loading," M.I.T. Pub. R62-113 (Soils Engr. Div.)

Whitman, R. V., Richardson, A. M., and Healy, K. A., "Time Lags in Pore Pressure Measurements," Proc. 5th International Conf. on Soil Mech. and Found. Engineering, Vol. 1, paper 1/69, 1961.

Whitman, R. V., Richardson, A. M., and Nasim, N. M., "The Response of Soils to Dynamic Loadings: Report 10 Strength of Saturated Fat Clay," Report by M.I.T. to U.S. Army Waterways Experiment Station, (in press) 1962.

Yamaguchi, H., "Strain Increments and Volume Change in the Plastic Flow of a Granular Material," Proc. Fifth International Conf. on Soil Mech. and Found. Engr., Paris, 1961, pp. 413-417.